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Appeals immune bargaining solution with variable alternative sets ☆

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Abstract

A bargaining solution based on the Rubinstein–Safra–Thomson 'ordinal Nash' outcome is investigated in the Peters–Wakker 'revealed group preferences' framework. Assuming non-expected utility preferences, necessary and sufficient conditions are stated on preference pairs in order for the solution to be well-defined and axiomatized uniquely.

JEL classification: C78

Keywords: Axiomatic Nash bargaining; Non-expected utility; Revealed group preferences

1. Introduction

Nash's axiomatic bargaining solution (Nash, 1950) assumes that the players are expected utility (EU) maximizers. Rubinstein, Safra and Thomson (RST) (Rubinstein et al., 1992) reinterpreted Nash's theory by introducing the ordinal Nash solution, which they characterized by axioms that refer to preference relations and physical alternatives. Rather than specifying the Nash outcome as one that maximizes a product of utilities, the ordinal Nash outcome is characterized as an outcome against which no player can successfully appeal, thus providing an interpretation of strategic interaction of two self-interested bargainers. RST extended the family of preference relations for which a unique ordinal Nash outcome exists beyond that of EU

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preferences. Given non-EU preferences, there exist bargaining problems for which no ordinal Nash outcome exists. Hanany and Safra (2000) presented necessary and sufficient conditions on players' preferences for existence and uniqueness of ordinal Nash outcomes.

The ordinal Nash solution methodology analyzed bargaining problems with varying individual preferences on a fixed set of alternatives. A different approach to the analysis of bargaining solutions, initiated by Peters and Wakker (1991) and followed by Ok and Zhou (2000), considers the agreements reached by players in different bargaining situations as a way to reveal the bargainers' preferences as a group. A group (not necessarily transitive) binary relation is said to represent a bargaining solution if for each bargaining problem, the corresponding bargaining outcomes are in group relation to all feasible alternatives. In this paper we take this approach by answering the following question: keeping the players' preferences fixed and varying the set of alternatives, what are the exact conditions on players' preferences that allow an axiomatization of a bargaining solution represented by a group binary relation according to the ordinal Nash approach? Denicolò (1996) axiomatized a solution that derives ordinal Nash outcomes when players' preferences remain fixed while the set of alternatives is allowed to vary. However, his analysis was restricted to a specific class of bargaining problems, where the two players have non-EU, rank dependent utility preferences (Quiggin, 1982 and Weymark, 1981) with identical probability distortion functions. In this paper we dispense with this rather uncomfortable initial assumption on players' preferences and instead derive it as a special case.

We consider the *biseparable* preferences proposed by Ghirardato and Marinacci (2001), by which lotteries whose supports contain at most two prizes are evaluated by probability distorted expected utility. These preferences include many well known non-EU families of preferences, e.g. rank-dependent utility (RDU) and disappointment aversion (Gul, 1991). Given the players' preferences, we define a group binary relation over a universal set of alternatives. Our first major result states necessary and sufficient conditions on preferences for the solution to be well-defined, i.e. non-empty valued, thus we derive exact conditions for existence. These conditions require some connection between the players' preferences, reflected by a joint restriction on the way they deviate from EU. In other words, it is crucial for existence of bargaining outcomes that players have some similarity in risk attitudes, although EU is much too restrictive as an initial assumption. Despite this similarity, the preferences are general enough to accommodate known violations like the 'Allais paradox' and the 'common ratio effect'. We also show that the bargaining solution is well-defined only when the group binary relation is transitive.

Over the set of preferences for which the solution is well-defined, the second main result shows that the solution outcome set is defined according to a Nash-like principle. More specifically, the outcome set maximizes the product of value gains over the disagreement outcome, referring to the value functions of the players' preference representations, when chosen appropriately to be normalized for probability distortion functions. Finally, in order to complete the presentation, the solution is characterized with similar axioms to those used by Denicolò, while being applicable to our wider set of preference relation pairs. The axioms include Pareto optimality, the weak axiom of revealed preferences and symmetry on symmetric problems over the set of *elementary* lotteries, i.e. lotteries over two alternatives, one of which is the disagreement outcome.

The paper is organized as follows. Section 2 presents the bargaining solution and analyses the conditions for which it is well-defined and Section 3 states the axiomatization of the solution for the sets of preferences investigated.

2. Appeals immune bargaining solution

We consider two-player bargaining games that are characterized by elements of the form $\langle \mathbf{X}, D, \succsim_1, \succsim_2 \rangle$. The set $\mathbf{X} \subseteq \mathbb{R}^2$ is a set of bargaining outcomes and $D \in \mathbb{R}^2$ is the disagreement outcome. The players' preference relations $\succsim_i$ $(i = 1, 2)$ are defined over the set of simple (finite) lotteries over $\mathbb{R}^2$, where for every $\mathbf{x} \in \mathbb{R}^2$, player i's preference relation depends only on the ith coordinate x_i. For simplicity of notation, a degenerate lottery with prize $\mathbf{x}$ is denoted by $\mathbf{x}$. Let $\mathcal{P}$ denote the set of all preference relations that are complete, transitive, continuous on the set of 2-prize lotteries of the form $p\mathbf{x} + (1 - p)\mathbf{y}$ (with respect to the topology of weak convergence) and strictly monotone with respect to the relation of first-order stochastic-dominance for 2-prize lotteries.[1] As usual, $\sim_i$ and $\succ_i$ denote the symmetric and asymmetric components of $\succsim_i$, respectively.

The domain $\mathcal{B}$ of bargaining games is defined so that the players' preferences $\succsim_i$ are fixed and the set of feasible alternatives is allowed to vary. As explained in the introduction, this conforms with the approach initiated by Peters and Wakker (1991), in which the agreements reached by players in different bargaining situations are used to reveal the bargainers' preferences as a group. Thus in the sequel, the pair of preferences is omitted from the notation of a bargaining problem. We define $\mathcal{B}$ as the set of all pairs $\langle \mathbf{X}, D \rangle$ which satisfy: (1) $\mathbf{X}$ is compact, (2) $\mathbf{X}$ is D-comprehensive, i.e. $D \in \mathbf{X}$ and for every $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $D \leqslant \mathbf{y} \leqslant \mathbf{x} \in \mathbf{X}$ implies[2] $\mathbf{y} \in \mathbf{X}$, (3) for every $\mathbf{x} \in \mathbf{X}$, $\mathbf{x} \geqslant D$, (4) there exists $\mathbf{x} \in \mathbf{X}$ such that $\mathbf{x} > D$ and (5) the efficient frontier of $\langle \mathbf{X}, D \rangle$, $F(\mathbf{X}, D) = \{\mathbf{x} \in \mathbf{X} \mid (\mathbf{y} \in \mathbf{X}) \wedge (\mathbf{y} \geqslant \mathbf{x}) \Rightarrow \mathbf{y} = \mathbf{x}\}$ is a connected set.

A bargaining solution specifies for each problem from its domain a subset of feasible alternatives as an outcome set.

Definition 1. For each pair of preference relations $\langle \succsim_1, \succsim_2 \rangle \in \mathcal{P}^2$, a bargaining solution $\mathcal{F}_{\langle \succsim_1, \succsim_2 \rangle}$ is a correspondence $\mathcal{F}_{\langle \succsim_1, \succsim_2 \rangle} : \mathcal{B} \to 2^{\mathbb{R}^2} \backslash \emptyset$ such that for any $\langle \mathbf{X}, D \rangle \in \mathcal{B}$, $\mathcal{F}_{\langle \succsim_1, \succsim_2 \rangle}(\mathbf{X}, D) \subseteq \mathbf{X}$.

The specific bargaining solution analyzed here is based on the strategic interaction interpretation of ordinal Nash outcomes, suggested by Rubinstein et al. (1992). For a given D, if $\mathbf{x} \succsim_i D$, $i = 1, 2$, a lottery giving $\mathbf{x}$ with probability p and D with $1 - p$ is called an *elementary* lottery and is denoted $p\mathbf{x}$. An outcome $\mathbf{x}$ is an *appeal against* $\mathbf{y}$, if there exist $i \in \{1, 2\}$ and $p \in [0, 1]$ such that $p\mathbf{x} \succ_i \mathbf{y}$ while $\mathbf{x} \succ_j p\mathbf{y}$ $(j \neq i)$. It is interpreted that both players perceive the probability of breakdown to be $1 - p$, player i is willing to take the risk of a possible breakdown when insisting on $\mathbf{x}$, while player j is unwilling to do so when insisting on $\mathbf{y}$. Under these conditions it is reasonable to conclude that i's appeal against the alternative $\mathbf{y}$ is successful, thus $\mathbf{y}$ is eliminated from the set of bargaining outcomes. We denote this property by $\mathbf{x} \succ \mathbf{y}$. Given a set of alternatives $\mathbf{X}$, we consider outcomes that are immune to all possible appeals in the bargaining problem $\langle \mathbf{X}, D \rangle$. Roughly speaking, such outcomes 'maximize' a group binary relation $\succsim$ over $\mathbf{X}$, defined by $\mathbf{x} \succsim \mathbf{y}$ if not $\mathbf{y} \succ \mathbf{x}$. The relation $\succsim$ is entitled the '*weak appeals relation*'. We also write $\mathbf{x} \sim \mathbf{y}$ when not $\mathbf{x} \succ \mathbf{y}$ and not $\mathbf{y} \succ \mathbf{x}$. Thus we may define the following based on the weak appeals relation.

[1] Formally, $\forall \mathbf{x}, \mathbf{y}, \forall p, q \in [0, 1]$, $x_i > y_i$ and $p > q$ imply $p\mathbf{x} + (1 - p)\mathbf{y} \succ_i q\mathbf{x} + (1 - q)\mathbf{y}$.

[2] We use the following notations: $\mathbf{x} \geqslant \mathbf{y} \Leftrightarrow \forall i, x_i \geqslant y_i$; $\mathbf{x} > \mathbf{y} \Leftrightarrow \forall i, x_i > y_i$.

Definition 2. Let $\langle \mathbf{X}, D\rangle \in \mathcal{B}$ be a bargaining problem. An outcome $\mathbf{y}^* \in \mathbf{X}$ is *immune to appeals* (*appeals-immune*) in $\langle \mathbf{X}, D\rangle$ if it is a maximal element of $\succsim$ over $\mathbf{X}$, i.e. for every $\mathbf{x} \in \mathbf{X}$, $\mathbf{y}^* \succsim \mathbf{x}$. That is,

$$\forall \mathbf{x} \in \mathbf{X},\ \forall p \in [0,1],\ \forall i, j \in \{1,2\},\ i \neq j, \quad p\mathbf{x} \succ_i \mathbf{y}^* \quad \Rightarrow \quad p\mathbf{y}^* \succsim_j \mathbf{x}.$$

Remark 1. A further strategic interaction interpretation for appeals-immune outcomes can be found in Burgos et al. (2002a, 2002b). Considering alternating offers bargaining games over a perfectly divisible object, with an exogenous probability of breakdown after a refusal in any round, they show that a stationary subgame perfect equilibrium (SSPE) outcome exists under general conditions on players' preferences. For problems having a unique SSPE, they show that in the limit, as the probability of breakdown goes to zero, the SSPE converges to an outcome satisfying a property they call 'equally marginally bold' (EMB) and moreover, any ordinal Nash outcome is an EMB. Thus the results of Burgos, Grant and Kajii imply that an appeals-immune outcome is the limit point of SSPE outcomes in division bargaining problems for which these outcomes exist uniquely.

Note that an appeals-immune outcome must belong to $F(\mathbf{X}, D)$, the efficient frontier of $\langle \mathbf{X}, D\rangle$. Also note that if $\mathbf{y}^*$ is appeals-immune then $\mathbf{y}^* > D$. This is true since $y_i^* \leqslant D_i$ implies that any $\mathbf{x}$ with $\mathbf{x} > D$ is an appeal (made by i) against $\mathbf{y}^*$.

When $\succsim_i$ are EU preferences, RST showed that an outcome maximizes the product of individual utility gains over the disagreement outcome, if, and only if, it is an appeals-immune outcome. Since such a maximizing outcome exists for all problems $\langle \mathbf{X}, D\rangle \in \mathcal{B}$, then for the EU case, the appeals-immune solution is well-defined on $\mathcal{B}$. However, in Hanany and Safra (2000), examples were given of bargaining problems with non-expected utility preferences, for which no appeals-immune outcome exists. In these cases, the bargaining solution cannot be represented by the weak appeals relation, since there exist sets of alternatives for which the outcome set is empty (see Example 1 below). Thus, we search for pairs of preference relations $\langle \succsim_1, \succsim_2\rangle \in \mathcal{P}^2$, such that for each $\langle \mathbf{X}, D\rangle \in \mathcal{B}$, an appeals-immune outcome exists and therefore the solution is well-defined. For each pair $\langle \succsim_1, \succsim_2\rangle$ which satisfies this property, we define the following.

Definition 3. The appeals-immune solution for $\langle \succsim_1, \succsim_2\rangle \in \mathcal{P}^2$ is a correspondence $\mathcal{A}_{\langle \succsim_1, \succsim_2\rangle}$: $\mathcal{B} \to 2^{\mathbb{R}^2} \setminus \emptyset$, such that for any $\langle \mathbf{X}, D\rangle \in \mathcal{B}$, $\mathcal{A}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D) \subseteq \mathbf{X}$ is the set of all appeals-immune outcomes of the bargaining problem $\langle \mathbf{X}, D\rangle$.[3]

The analysis of conditions on players' preferences that ensure a well-defined solution is carried out by investigating the connection between the appeals-immune solution and the weak

[3] An appeals-immune outcome may be dominated by a lottery. This is the case, for example, when the preference relations are EU while the deterministic outcome set is not convex in the von Neumann–Morgenstern (vNM) utility space. One may suggest that a bargaining solution should require that for any $\langle \mathbf{X}, D\rangle \in \mathcal{B}$, all solution outcomes are non-dominated by any lottery. However, this solution would not be well-defined even for the domain of EU preferences. Indeed, for every pair of EU preferences, there exists $\langle \mathbf{X}, D\rangle$ for which the set of deterministic outcomes in the vNM utility space is not convex and all deterministic outcomes are dominated, thus the solution outcome set is empty. A similar conclusion holds if the solution is restricted such that for any $\langle \mathbf{X}, D\rangle$, at least one solution outcome must be non-dominated. Since we are interested in extensions of the Nash solution, which therefore should be well-defined for the EU case, attention is restricted to a solution that does not require the lottery non domination property, that is, the appeals-immune solution.

appeals relation. Specifically, the transitivity property of the weak appeals relation turns out to be of crucial importance with respect to the definition of the appeals-immune solution. This conclusion is shown in the following proposition, which is in fact a general result on the existence of maximal elements in an ordered set, that is useful for our analysis.

Proposition 1. *Let $D \in \mathbb{R}^2$ and suppose that a relation $\succsim$ over $\{\mathbf{x} \in \mathbb{R}^2 \mid \mathbf{x} \geqslant D\}$ is complete, continuous and monotone with respect to $\geqslant$. Then $\succsim$ is transitive if, and only if, for any $\mathbf{X}$ such that $\langle \mathbf{X}, D\rangle \in \mathcal{B}$, there exists a maximal element of $\succsim$ over $\mathbf{X}$.*

Proof. Suppose that $\succsim$ is transitive. Since $\mathbf{X}$ is compact and the relation $\succsim$ is complete and continuous over $\mathbb{R}^2$, there exists a non-empty subset $\mathbf{Y} \subseteq \mathbf{X}$ such that each $\mathbf{x} \in \mathbf{Y}$ maximizes the preference relation $\succsim$ over $\mathbf{X}$, which completes the 'only if' part of the proof.

For the 'if' part, suppose that $\succsim$ is non-transitive. Thus, there exist $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^2$ such that $\mathbf{x} \succsim \mathbf{y} \succsim \mathbf{z}$ but $\mathbf{z} \succ \mathbf{x}$. Since the relation $\succsim$ is continuous, there exist $\mathbf{x}', \mathbf{y}', \mathbf{z}'$ such that $\mathbf{x}' \succ \mathbf{y}' \succ \mathbf{z}' \succ \mathbf{x}'$. Let $\mathbf{Y} = \{\mathbf{w} \mid (D \leqslant \mathbf{w} \leqslant \mathbf{x}') \vee (D \leqslant \mathbf{w} \leqslant \mathbf{y}') \vee (D \leqslant \mathbf{w} \leqslant \mathbf{z}')\}$. For any $\mathbf{w}, \mathbf{w}' \in \mathbf{Y}$ such that $\mathbf{w} \neq \mathbf{w}'$ and $\mathbf{w}' \geqslant \mathbf{w}$, clearly $\mathbf{w}' \succ \mathbf{w}$. Thus, there exist no maximizing elements in $\mathbf{Y}$. The set $\langle \mathbf{Y}, D\rangle \notin \mathcal{B}$, since the efficient frontier of $\mathbf{Y}$, $F(\mathbf{Y}, D) \subseteq \{\mathbf{x}', \mathbf{y}', \mathbf{z}'\}$, is not a connected set. However, by continuity and monotonicity of the relation $\succsim$, there exists $\mathbf{X} \supseteq \mathbf{Y}$ such that $\langle \mathbf{X}, D\rangle \in \mathcal{B}$ and for each $\mathbf{w} \in \mathbf{X}\backslash\mathbf{Y}$, there exists $\mathbf{w}' \in \{\mathbf{x}', \mathbf{y}', \mathbf{z}'\}$ such that $\mathbf{w}' \succ \mathbf{w}$ (see Fig. 1). Hence, there are no maximizing elements of $\succsim$ in $\mathbf{X}$. $\square$

The result drawn from Proposition 1 is powerful, since it states that transitivity of the weak appeals relation is necessary and sufficient for the appeals-immune solution to be well-defined. In order to use this result to analyze extensions to non-expected utility, we define the notion of induced utilities similarly to Hanany and Safra (2000). This definition allows a useful connection between the weak appeals relation and the players' induced utilities, as shown in Lemma 1 below. Given a disagreement outcome D, let $\mathcal{U}_i(D)$ be the set of all continuous functions $u_i : \{t \in \mathbb{R} \mid t \geqslant D_i\} \times \{t \in \mathbb{R} \mid t > D_i\} \to \mathbb{R}_+$ that increase in the first argument, decrease in the second, satisfy $u_i(t;t) = 1$, $u_i(D_i;t) = 0$ and $u_i(s;t)u_i(t;s) = 1$.

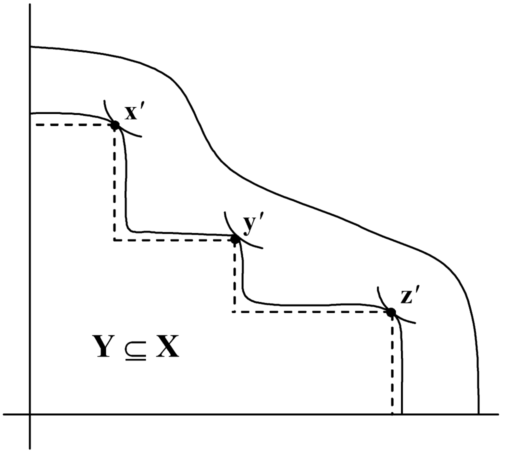

Fig. 1. Existence of $\langle \mathbf{X}, D\rangle \in \mathcal{B}$ with no maximizing elements of $\succsim$.

Definition 4. Let $D \in \mathbb{R}^2$. The ith induced utility mapping is the function $IU_i : \mathcal{P} \to \mathcal{U}_i(D)$, that is defined by

$$IU_i(\succsim_i)(x_i; y_i) = \begin{cases} p & \text{if } x_i \sim_i p y_i, \\ \frac{1}{p} & \text{if } y_i \sim_i p x_i. \end{cases}$$

The function $u_i = IU_i(\succsim_i)$ is the *induced utility* of $\succsim_i$. In Hanany and Safra (2000) it is shown that the induced utilities u_i are well-defined and satisfy

$$p\mathbf{x} \sim_i \mathbf{y} \iff p u_i(x_i; x_i) = u_i(y_i; x_i) \iff p u_i(x_i; y_i) = u_i(y_i; y_i).$$

For an EU preference relation with utility function v_i, $u_i(x_i; y_i) = \frac{v_i(x_i) - v_i(D_i)}{v_i(y_i) - v_i(D_i)}$. The next lemma is useful as an intermediate step before analyzing the transitivity of $\succsim$.

Lemma 1. *Let $D \in \mathbb{R}^2$. The weak appeals relation $\succsim$ satisfies the following for any $\mathbf{x}, \mathbf{y} \geqslant D$:*

(1) $\mathbf{y} > D$ *implies* $\mathbf{x} \succsim \mathbf{y} \Leftrightarrow \prod_i u_i(x_i; y_i) \geqslant 1$,
(2) *not* $\mathbf{y} > D$ *implies* $\mathbf{x} \succsim \mathbf{y}$,
(3) *not* $\mathbf{x} > D$ *and* $\mathbf{x} \succsim \mathbf{y}$ *implies not* $\mathbf{y} > D$.

Furthermore, the relation $\succsim$ is complete, continuous and monotone with respect to $\geqslant$ over $\{\mathbf{x} \in \mathbb{R}^2 \mid \mathbf{x} \geqslant D\}$.

Proof. Let $u_i(\cdot;\cdot)$ be the induced utility functions of $\succsim_i$. To show (2), if not $\mathbf{y} > D$ then there exist a player k for which $\mathbf{y} \sim_k D$, thus for any $\mathbf{x}$, $p \in [0, 1]$, $p\mathbf{y} \succ_i \mathbf{x}$ and $\mathbf{y} \succ_j p\mathbf{x}$ cannot both hold and therefore $\mathbf{x} \succsim \mathbf{y}$. To show (1), if $\mathbf{y} > D$ then,

$$\begin{aligned} &\mathbf{x} \succsim \mathbf{y} \\ &\iff \text{not } \mathbf{y} \succ \mathbf{x} \\ &\iff \forall i, \forall p \in [0,1],\ p\mathbf{y} \succ_i \mathbf{x} \Rightarrow p\mathbf{x} \succsim_j \mathbf{y}\ (j \neq i) \\ &\iff \forall i, \forall p \in [0,1],\ p > u_i(x_i; y_i) \Rightarrow p \geqslant u_j(y_j; x_j) \\ &\iff \forall i,\ u_i(x_i; y_i) \geqslant u_j(y_j; x_j) \\ &\iff \prod_i u_i(x_i; y_i) \geqslant 1. \end{aligned}$$

To see why the condition $\forall i$, $\forall p \in [0, 1]$, $p > u_i(x_i; y_i) \Rightarrow p \geqslant u_j(y_j; x_j)$ implies $\forall i$, $u_i(x_i; y_i) \geqslant u_j(y_j; x_j)$, note that $u_i(x_i; y_i) \geqslant u_j(y_j; x_j) \Leftrightarrow u_j(x_j; y_j) \geqslant u_i(y_i; x_i)$; if $\mathbf{x} \geqslant \mathbf{y}$ then $\forall i$, $u_i(x_i; y_i) \geqslant 1 \geqslant u_j(y_j; x_j)$; if $x_i < y_i$ and $u_i(x_i; y_i) < u_j(y_j; x_j)$ then there exists $p \in [0, 1]$ such that $u_i(x_i; y_i) < p < u_j(y_j; x_j)$. To show (3), if not $\mathbf{x} > D$ and $\mathbf{x} \succsim \mathbf{y}$ then not $\mathbf{y} > D$, otherwise $\prod_i u_i(x_i; y_i) = 0$ and thus not $\mathbf{x} \succsim \mathbf{y}$, a contradiction. Completeness follows by (2) and since for $\mathbf{y} > D$ either $\prod_i u_i(x_i; y_i) \geqslant 1$ or $\prod_i u_i(x_i; y_i) \leqslant 1$, which implies $\prod_i u_i(y_i; x_i) \geqslant 1$. Continuity of the weak appeals relation $\succsim$ follows from the continuity of $\succsim_i$ on the set of elementary lotteries. Monotonicity follows since $\mathbf{x} \geqslant \mathbf{y}$ implies $\prod_i u_i(x_i; y_i) \geqslant 1$, thus $\mathbf{x} \succsim \mathbf{y}$. $\square$

Lemma 1 states that the weak appeals relation is complete, but does not refer to it being transitive. For EU or DL preferences, $\succsim$ is clearly transitive, since it is represented by the product of individual utility gains over the disagreement outcome. Unfortunately, there exist examples of preference pairs in $\mathcal{P}^2$, for which the appeals-immune relation is not transitive, as demonstrated in the example below. The following is needed to state the example. Let $\mathcal{P}^B \subseteq \mathcal{P}$ be the set of *biseparable* preference relations, proposed by Ghirardato and Marinacci (2001). For these preferences, the value of 2-prize lotteries $p\mathbf{x} + (1-p)\mathbf{y}$ for which $\mathbf{x} \succsim_i \mathbf{y}$ is given by $g_i(p)v_i(x_i) + [1 - g_i(p)]v_i(y_i)$, where g_i and v_i are strictly monotone, $g_i : [0,1] \to [0,1]$ is onto and determined uniquely and v_i is unique up to positive affine transformations. The preference set $\mathcal{P}^B$ contains the set of EU preference relations as well as the set of DL preferences introduced by Grant and Kajii (1995) (in both cases $g_i(p) = p \ \forall p \in [0,1]$). The set $\mathcal{P}^B$ also includes the family of rank-dependent utility (RDU) preferences [see Quiggin (1982) and Weymark (1981)] and Gul (1991) disappointment aversion (DA) family. Note that for any $\succsim_i \in \mathcal{P}^B$,

$$u_i(x_i; y_i) = \begin{cases} g_i^{-1}\left(\frac{v_i(x_i)}{v_i(y_i)}\right) & \text{if } y_i \geqslant x_i, \\ \frac{1}{g_i^{-1}\left(\frac{v_i(y_i)}{v_i(x_i)}\right)} & \text{if } x_i > y_i. \end{cases}$$

Example 1. Let $\langle \succsim_1, \succsim_2 \rangle \in (\mathcal{P}^B)^2$, where $\succsim_1$ is a risk neutral EU preference with a vNM utility function $v_1(x_1) = x_1$ and $\succsim_2$ is a DA preference represented such that $v_2(x_2) = x_2$ and $g_2(p) = \frac{p}{1+(1-p)\beta_2}$, $\beta_2 = 1$. The corresponding induced utility functions satisfy $u_1(x_1; y_1) = \frac{x_1}{y_1}$ and $u_2(x_2; y_2) = \frac{2}{1+y_2/x_2}$ for $\mathbf{y} \succsim_2 \mathbf{x}$. Let $\mathbf{X} = \{\mathbf{x} \mid x_i \geqslant 0, \ \sum_i x_i \leqslant 9\}$ and $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbf{X}$, where $\mathbf{x} = (6,1)$, $\mathbf{y} = (3,3)$, $\mathbf{z} = (2,6)$ and $D = (0,0)$. Then, $\prod_i u_i(x_i; y_i) = \frac{6}{3}\frac{2}{1+3/1} = 1$, $\prod_i u_i(y_i; z_i) = \frac{3}{2}\frac{2}{1+6/3} = 1$ and $\prod_i u_i(x_i; z_i) = \frac{6}{2}\frac{2}{1+6/1} < 1$. Therefore, by Lemma 1, $\mathbf{x} \sim \mathbf{y} \sim \mathbf{z}$ but $\mathbf{z} \succ \mathbf{x}$ (see Fig. 2). By Proposition 1, no bargaining solution can be represented by the weak appeals relation corresponding to $\langle \succsim_1, \succsim_2 \rangle$.

This example motivates the investigation of pairs of biseparable preferences for which $\succsim$ is transitive. In the example, the individuals have different probability distortion functions. It turns out that this fact is crucial to transitivity, as can be seen by Proposition 2. The result proves a

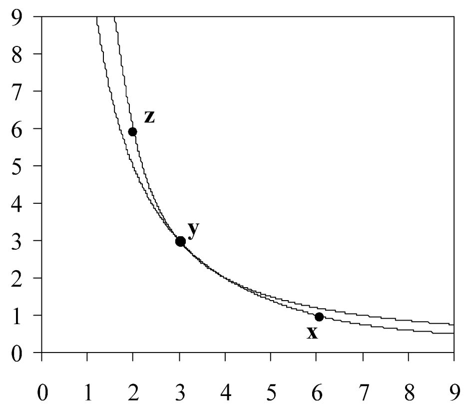

Fig. 2. Non-transitive weak appeals indifference curves.

necessary[4] and sufficient condition for transitivity, stated on the distortion functions g_i, not to the functions v_i, in the biseparable representations corresponding to $\succsim_i$.

Proposition 2. *Let $\langle \succsim_1, \succsim_2 \rangle \in (\mathcal{P}^B)^2$, $D \in \mathbb{R}^2$ and let $\succsim$ be the corresponding weak appeals relation. Then, $\succsim$ is transitive if, and only if, $\succsim_i$ have biseparable representations such that $g_1 = (g_2)^\delta$, for some $\delta > 0$.*

Proof. Suppose that $\succsim_i$ have biseparable representations such that $g_1 = (g_2)^\delta$ for some $\delta > 0$, where the value functions are denoted by $v_1, \tilde{v}_2$. Then normalizing $v_i(D_i) = 0$ and letting $v_2 = (\tilde{v}_2)^{1/\delta}$, $\succsim_i$ can be represented for any $p\mathbf{x}$ by $g(p)v_i(x_i)$, where $g = g_1$. Let $\mathbf{x}, \mathbf{y}, \mathbf{z}$ such that $\mathbf{x} \succsim \mathbf{y} \succsim \mathbf{z}$. If $\mathbf{x} \geqslant \mathbf{z}$ or not $\mathbf{x} > D$ or not $\mathbf{y} > D$ or not $\mathbf{z} > D$, then $\mathbf{x} \succsim \mathbf{z}$ by Lemma 1. Otherwise, either $v_i(x_i) \geqslant v_i(y_i)$ for each i, or there exist i, j $(i \neq j)$ such that $g^{-1}(\frac{v_i(y_i)}{v_i(x_i)}) = u_i(y_i; x_i) \leqslant u_j(x_j; y_j) = g^{-1}(\frac{v_j(x_j)}{v_j(y_j)})$ and thus $\prod_k \frac{v_k(x_k)}{v_k(y_k)} \geqslant 1$. Similarly, $\prod_k \frac{v_k(y_k)}{v_k(z_k)} \geqslant 1$ and therefore, $\prod_k \frac{v_k(x_k)}{v_k(z_k)} \geqslant 1$. Let $i \neq j$ such that $x_i \leqslant z_i$ and $x_j \geqslant z_j$. Then $u_i(x_i; z_i) = g^{-1}(\frac{v_i(x_i)}{v_i(z_i)}) \geqslant g^{-1}(\frac{v_j(z_j)}{v_j(x_j)}) = u_j(z_j; x_j)$, thus $\prod_i u_i(x_i; z_i) \geqslant 1$, therefore $\mathbf{x} \succsim \mathbf{z}$ by Lemma 1. Hence $\succsim$ is transitive.

To prove the converse, suppose that for both i, $\succsim_i$ is represented for any $p\mathbf{x}$ by $g_i(p)v_i(x_i)$, where $v_i(D_i) = 0$. Let $f : [0,1] \to [0,1]$ such that $\forall \alpha \in [0,1]$, $f(\alpha) = g_1[g_2^{-1}(\alpha)]$. Let $\alpha_1, \beta_1 \in [0,1]$, $\alpha_2 = f(\alpha_1)$, $\beta_2 = f(\beta_1)$ and $D < \mathbf{w} \in \mathbf{X}$. Then, by continuity and monotonicity of both v_i, $v_i(D_i) = 0$ and $\mathbf{X}$ being D-comprehensive, there exist $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbf{X}$ such that $x_2 = w_2$, $v_1(x_1) = \alpha_2\beta_2 v_1(w_1)$, $v_1(y_1) = \alpha_2 v_1(w_1)$, $v_2(y_2) = \beta_1 v_2(w_2)$, $z_1 = w_1$ and $v_2(z_2) = \alpha_1\beta_1 v_2(w_2)$. Therefore $u_2(z_2; y_2) = g_2^{-1}(\alpha_1) = g_1^{-1}(\alpha_2) = u_1(y_1; z_1)$ and $u_2(y_2; x_2) = g_2^{-1}(\beta_1) = g_1^{-1}(\beta_2) = u_1(x_1; y_1)$, thus $\mathbf{x} \sim \mathbf{y} \sim \mathbf{z}$. Then, transitivity implies $\mathbf{x} \sim \mathbf{z}$, thus $g_2^{-1}(\alpha_1\beta_1) = u_2(z_2; x_2) = u_1(x_1; z_1) = g_1^{-1}(\alpha_2\beta_2)$ by Lemma 1. Therefore we get Cauchy's power functional equation $\forall \alpha_1, \beta_1 \in [0,1]$, $f(\alpha_1\beta_1) = f(\alpha_1)f(\beta_1)$, for which the unique solution $f(\cdot)$ that is continuous and strictly monotonically increasing, satisfies for some $\delta > 0$ and $\forall \alpha \in [0,1]$, $f(\alpha) = \alpha^\delta$. Hence, $\forall p \in [0,1]$, $g_1(p) = f[g_2(p)] = [g_2(p)]^\delta$. $\square$

Given Proposition 2, from now on we restrict attention to cases where $\succsim_i$ are taken from the set $\mathcal{P}^B$. The proposition allows the characterization of transitivity of the weak appeals relation based on a joint property of the players' preferences.

Definition 5 *(**UD**: Uniform Distortion).* A pair of preference relations $\langle \succsim_1, \succsim_2 \rangle \in (\mathcal{P}^B)^2$ satisfies **UD** if there exist $D \in \mathbb{R}^2$ and an increasing and onto function $\Psi_D : \{t \in \mathbb{R} \mid t \geqslant D_1\} \to \{t \in \mathbb{R} \mid t \geqslant D_2\}$ such that for every $p, q \in [0,1]$ and $x_1, y_1 \geqslant D_1$,

$$\begin{aligned} & px_1 + (1-p)D_1 \succsim_1 qy_1 + (1-q)D_1 \\ \iff \quad & p\Psi_D(x_1) + (1-p)D_2 \succsim_2 q\Psi_D(y_1) + (1-q)D_2. \end{aligned}$$

The set of all preference pairs satisfying **UD** is denoted by $\mathcal{D}^{UD}$.

[4] Note that the necessity of this condition is proved as a result of the requirement that the appeals indifference relation $\sim$ is transitive. Nevertheless this condition is sufficient for the transitivity of the whole weak appeals relation $\succsim$.

The set $\mathcal{D}^{UD}$is the set of all preference pairs in $(\mathcal{P}^B)^2$ for which the players' preferences have biseparable representations with equal probability distortion functions up to a positive power transformation, i.e. the condition in Proposition 2, $g_1 = (g_2)^\delta$ for $\delta > 0$. Indeed, assuming **UD**, if the biseparable representation of $\succsim_2$ is chosen such that $v_2(D_2) = 0$ and $\tilde{v}_1$ is defined such that $\forall x_1 \geqslant D_1$, $\tilde{v}_1(x_1) = v_2[\Psi_D(x_1)]$, then $\succsim_1$ can be represented for any elementary lottery $p\mathbf{x}$ by $g_2(p)\tilde{v}_1(x_1)$ and consequently by $[g_2(p)]^\delta[\tilde{v}_1(x_1)]^\delta$ for any $\delta > 0$. The uniqueness of the distortion function g_i for preferences in $\mathcal{P}^B$ then implies $g_1 = (g_2)^\delta$ for some $\delta > 0$. Similarly, the latter condition implies **UD** by defining Ψ_D such that $v_1(x_1) = [v_2[\Psi_D(x_1)]]^\delta$. Moreover, for pairs of preferences in $\mathcal{D}^{UD}$, it is possible to evaluate any $p\mathbf{x}$ by $g(p)v_i(x_i)$, i.e. with *identical* distortion functions g for both i. This can be achieved by taking an appropriate power transformation that preserves the representation for elementary lotteries and makes both distortion functions identical. Indeed, given biseparable representations of $\succsim_i$ such that $v_i(D_i) = 0$, let $\tilde{v}_2 = (v_2)^\delta$, then $\succsim_2$ can be represented for any $p\mathbf{x}$ by $[g_2(p)]^\delta[v_2(x_2)]^\delta$, i.e. by $g_1(p)\tilde{v}_2(x_2)$. Therefore an equivalent way to define the uniform distortion property is the requirement that both players have identical distortion functions in *some* representation when considering *only* elementary lotteries. Note that there are pairs in $(\mathcal{P}^B)^2\backslash\mathcal{D}^{UD}$ that do not satisfy the property **UD**, consequently the weak appeals relation is not transitive as shown in Example 1.

Following are examples for known families of preferences for which the intersection with the set $\mathcal{D}^{UD}$ is non-empty.

Example 2. (1) EU and DL preferences. The set $\mathcal{D}^{UD}$ contains the set of all pairs of DL preferences (Grant and Kajii, 1995), i.e. preferences that are represented with $g(p) = p$, $\forall p \in [0, 1]$. This set contains the set of EU preference pairs.

(2) RDU preferences. The set $\mathcal{D}^{UD}$ contains the set of all pairs of RDU preferences (Quiggin, 1982 and Weymark, 1981) having representations such that $g_1 = (g_2)^\delta$ for $\delta > 0$, i.e. equal distortion functions up to a positive power transformation. This set allows for a very wide range of risk attitudes, including behavior which is consistent with the 'Allais paradox' and the 'common ratio effect' (such behavior is excluded by DL and EU preferences). Given a pair of preferences, the difference between the players is characterized by the power transformation parameter δ, as well as by the entirely unrestricted curvature of the outcome value functions v_i. Since the distortion function g in the representation of a RDU preference is unique, a positive power transformation preserves the **UD** property for elementary lotteries but fails to maintain the preference representation for general lotteries. This demonstrates that the restriction implemented by **UD** is weak, in the sense that only preferences over elementary lotteries are involved. Moreover, the requirement that both players have identical RDU distortion functions for general lotteries is much more restrictive than the property **UD**.

(3) DA preferences. The set $\mathcal{D}^{UD}$ contains the set of all pairs of DA preferences (Gul, 1991) represented with $g_i(p) = \frac{p}{1+(1-p)\beta}$ for some common $\beta > 0$. Example 1 demonstrates why it is essential that the DA parameter β is equal for both players in order for the solution to be well-defined.

Proposition 2 states that the weak appeals relation is transitive only for all the profiles in the set $\mathcal{D}^{UD}$. Extending this result further, the following theorem states that the appeals-immune solution $\mathcal{A}_{\langle\succsim_1,\succsim_2\rangle}(\cdot)$ is well-defined only for $\langle\succsim_1,\succsim_2\rangle$ in the set $\mathcal{D}^{UD}$. Moreover, in this case it has a Nash-like utility product representation.

Theorem 1. (1) *The appeals-immune solution for* $\langle \succsim_1, \succsim_2 \rangle \in (\mathcal{P}^B)^2$, $\mathcal{A}_{\langle \succsim_1, \succsim_2 \rangle}$, *is well-defined on* $\mathcal{B}$ *if, and only if,* $\langle \succsim_1, \succsim_2 \rangle \in \mathcal{D}^{UD}$.

(2) *Let* $\langle \succsim_1, \succsim_2 \rangle \in \mathcal{D}^{UD}$. *Then, for any* $\langle \mathbf{X}, D \rangle \in \mathcal{B}$, *the appeals-immune solution is defined by* $\mathcal{A}_{\langle \succsim_1, \succsim_2 \rangle}(\mathbf{X}, D) = \arg\max_{\mathbf{x} \in \mathbf{X}} \prod_i v_i(x_i)$, *where* v_i *are chosen such that* $\succsim_i$ *is represented for elementary lotteries* $p\mathbf{x}$ *by* $g(p)v_i(x_i)$ *for a common probability distortion function* g.

Proof. (1) This follows immediately from Lemma 1 and Propositions 1 and 2.

(2) Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$. If not $\mathbf{y} > D$, then $\prod_i v_i(x_i) \geqslant \prod_i v_i(y_i) = 0$ and $\mathbf{x} \succsim \mathbf{y}$ by Lemma 1. If $\mathbf{x}, \mathbf{y} > D$, then $\mathbf{x} \succsim \mathbf{y}$ if, and only if, $\prod_i u_i(x_i; y_i) \geqslant 1$ by Lemma 1, which holds if, and only if, there exist i, j $(i \neq j)$ such that $g^{-1}[\frac{v_j(x_j)}{v_j(y_j)}] = u_j(x_j; y_j) \geqslant u_i(y_i; x_i) = g^{-1}[\frac{v_i(y_i)}{v_i(x_i)}]$. This is the case if, and only if, $\prod_i v_i(x_i) \geqslant \prod_i v_i(y_i)$. By definition, for any $\mathbf{y}^* \in \mathcal{A}_{\langle \succsim_1, \succsim_2 \rangle}(\mathbf{X}, D)$ and $\mathbf{x} \in \mathbf{X}$, $\mathbf{y}^* \succsim \mathbf{x}$. Hence $\mathbf{y}^* \in \mathcal{A}_{\langle \succsim_1, \succsim_2 \rangle}(\mathbf{X}, D)$ if, and only if, $\mathbf{y}^* \in \arg\max_{\mathbf{x} \in \mathbf{X}} \prod_i v_i(x_i)$ and therefore $\mathcal{A}_{\langle \succsim_1, \succsim_2 \rangle}(\mathbf{X}, D) = \arg\max_{\mathbf{x} \in \mathbf{X}} \prod_i v_i(x_i)$. □

The theorem presents the largest subset of $(\mathcal{P}^B)^2$ for which the appeals-immune solution is well-defined on the domain $\mathcal{B}$. For this subset, the theorem states that the information provided by the value functions v_i, chosen appropriately for the preference relations $\succsim_i$ to be normalized for probability distortion functions, is sufficient alone to derive the solution outcome set. This conclusion holds despite the fact that the value functions v_i generally convey only partial information about the preferences in $\mathcal{P}^B$ over the set of elementary lotteries. The result is due to the biseparable property of the preference relations in $\mathcal{P}^B$ and the **UD** property, which is required for the solution to be well-defined over its domain.

3. Axiomatic characterization

In this section we provide a characterization of the appeals-immune solution along the lines of Denicolò (1996). The axiomatization shown here applies to all cases where the solution is well-defined,[5] as characterized in the previous section, i.e. to preference relation pairs $\langle \succsim_1, \succsim_2 \rangle \in \mathcal{D}^{UD}$. The first two axioms represent the standard Pareto optimality condition and the weak axiom of revealed preferences.

Definition 6 *(Axiom **PAR**: Pareto Optimality).* For any bargaining problem $\langle \mathbf{X}, D \rangle \in \mathcal{B}$, $\mathcal{A}_{\langle \succsim_1, \succsim_2 \rangle}(\mathbf{X}, D) \subseteq F(\mathbf{X}, D)$.

Definition 7 *(Axiom **WARP**: Weak Axiom of Revealed Preferences).* For any bargaining problems $\langle \mathbf{X}, D \rangle$, $\langle \mathbf{Y}, D \rangle \in \mathcal{B}$ such that $\mathbf{Y} \subseteq \mathbf{X}$, if $\mathcal{A}_{\langle \succsim_1, \succsim_2 \rangle}(\mathbf{X}, D) \cap \mathbf{Y} \neq \emptyset$, then $\mathcal{A}_{\langle \succsim_1, \succsim_2 \rangle}(\mathbf{Y}, D) = \mathcal{A}_{\langle \succsim_1, \succsim_2 \rangle}(\mathbf{X}, D) \cap \mathbf{Y}$.

The third axiom defines symmetry, for which the following definition is required.

Definition 8. The bargaining problem $\langle \mathbf{X}, D \rangle$ is symmetric if there exists a function $\phi_{\mathbf{X}} : \mathbf{X} \to \mathbf{X}$, such that for every $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\phi_{\mathbf{X}}(\mathbf{x}) = \mathbf{y}$ implies $\phi_{\mathbf{X}}(\mathbf{y}) = \mathbf{x}$, $\phi_{\mathbf{X}}(D) = D$ and for every $p\mathbf{x}, q\mathbf{y}$,

[5] The analysis of the previous section shows that Denicolò's restriction to RDU preferences with $g_1 = g_2$ covers an extremely small subset of the cases where the solution is well-defined.

$p\mathbf{x} \succsim_1 q\mathbf{y} \Leftrightarrow p\phi_{\mathbf{X}}(\mathbf{x}) \succsim_2 q\phi_{\mathbf{X}}(\mathbf{y})$. The function $\phi_{\mathbf{X}}$ is then called a symmetry function of the bargaining problem $\langle \mathbf{X}, D\rangle$.

Definition 9 *(Axiom **SYM**: Symmetry).* For any symmetric bargaining problem $\langle \mathbf{X}, D\rangle \in \mathcal{B}$ with a symmetry function $\phi_{\mathbf{X}}$, $\mathbf{x} \in \mathcal{A}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D)$ if, and only if, $\phi_{\mathbf{X}}(\mathbf{x}) \in \mathcal{A}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D)$.

A symmetric problem is defined here in a different manner to that stated by Rubinstein et al. (1992) and subsequently followed by Denicolò, but instead it follows Grant and Kajii (1995). The axiom allows a problem to be symmetric based on the preference information over the set of elementary lotteries alone. This is justified by the definition of the appeals-immune solution, which is itself based only on that information.[6]

Theorem 2 characterizes the appeals-immune solution for the set $\mathcal{D}^{UD}$, the maximal subset of $(\mathcal{P}^B)^2$ for which the solution is well-defined over the domain $\mathcal{B}$, as stated in Theorem 1 (the proof is in Appendix A).

Theorem 2. *Let $\langle \succsim_1, \succsim_2\rangle \in \mathcal{D}^{UD}$ be a pair of preference relations. The appeals-immune solution $\mathcal{A}_{\langle \succsim_1, \succsim_2\rangle}(\cdot)$ is the unique bargaining solution which satisfies **PAR**, **WARP** and **SYM** over the domain $\mathcal{B}$.*

Appendix A

Proof of Theorem 2. Let v_i and g be chosen such that $\succsim_i$ is represented for elementary lotteries $p\mathbf{x}$ by $g(p)v_i(x_i)$, where $v_i(D_i) = 0$. First we will show that $\mathcal{A}_{\langle \succsim_1, \succsim_2\rangle}(\cdot)$ satisfies the axioms. Clearly **PAR** is satisfied since for any $\langle \mathbf{X}, D\rangle \in \mathcal{B}$ and $\mathbf{y}^* \in \mathcal{A}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D)$, $\mathbf{y}^* \in F(\mathbf{X}, D)$. Let $\langle \mathbf{X}, D\rangle$, $\langle \mathbf{Y}, D\rangle \in \mathcal{B}$ such that $\mathbf{Y} \subseteq \mathbf{X}$ and $\mathcal{A}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D) \cap \mathbf{Y} \neq \emptyset$, then $\max_{\mathbf{x}\in\mathbf{X}} \prod_i v_i(x_i) \geqslant \max_{\mathbf{x}\in\mathbf{Y}} \prod_i v_i(x_i)$. According to Theorem 1, $\max_{\mathbf{x}\in\mathbf{X}} \prod_i v_i(x_i) \leqslant \max_{\mathbf{x}\in\mathbf{Y}} \prod_i v_i(x_i)$, otherwise $\mathcal{A}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D) \cap \mathbf{Y} = \emptyset$, a contradiction. Thus $\arg\max_{\mathbf{x}\in\mathbf{Y}} \prod_i v_i(x_i) = \mathbf{Y} \cap \arg\max_{\mathbf{x}\in\mathbf{X}} \prod_i v_i(x_i)$, therefore **WARP** is satisfied. Suppose now that the problem $\langle \mathbf{X}, D\rangle$ is symmetric with a symmetry function ϕ. Since v_i are unique up to multiplication by a positive scalar, we can choose v_i such that $\max_{\mathbf{x}\in\mathbf{X}} v_1(x_1) = \max_{\mathbf{x}\in\mathbf{X}} v_2(x_2)$. Due to the symmetry of the problem, for every $\mathbf{x} \in \mathbf{X}$ and both i and $j \neq i$, $v_i(x_i) = v_j[(\phi(\mathbf{x}))_j]$. Thus, for every $\mathbf{x} \in \mathbf{X}$, $\prod_i v_i(x_i) = \prod_i [(\phi(\mathbf{x}))_i]$, therefore $\mathbf{y}^* \in \arg\max_{\mathbf{x}\in\mathbf{X}} \prod_i v_i(x_i)$ implies $\phi(\mathbf{y}^*) \in \arg\max_{\mathbf{x}\in\mathbf{X}} \prod_i v_i(x_i)$. Hence **SYM** is satisfied by Theorem 1.

To prove the converse, suppose that $\mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\cdot)$ is a bargaining solution that satisfies **PAR**, **WARP** and **SYM**. Let $\langle \mathbf{X}, D\rangle \in \mathcal{B}$ and suppose $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\mathbf{x} \neq \mathbf{y}$, satisfy $\prod_i v_i(x_i) = \prod_i v_i(y_i)$ and v_i are chosen such that $\max_i\{v_i(x_i)\} = \max_i\{v_i(y_i)\}$. Thus for both i and $j \neq i$, $v_i(x_i) = v_j(y_j)$. For any $\langle \mathbf{Y}, D\rangle \in \mathcal{B}$, define $V(\mathbf{Y}) = \{\langle v_1(w_1), v_2(w_2)\rangle \mid \mathbf{w} \in \mathbf{Y}\}$. Let $\tilde{\mathbf{Y}} = \{\mathbf{w} \mid (D \leqslant \mathbf{w} \leqslant \mathbf{x}) \vee (D \leqslant \mathbf{w} \leqslant \mathbf{y})\}$. Clearly $\tilde{\mathbf{Y}} \subseteq \mathbf{X}$ and any $(a, b) \in V(\tilde{\mathbf{Y}})$ implies $(b, a) \in V(\tilde{\mathbf{Y}})$. Note that $\langle \tilde{\mathbf{Y}}, D\rangle \notin \mathcal{B}$ since the efficient frontier of $\tilde{\mathbf{Y}}$, $F(\tilde{\mathbf{Y}}, D) = \{\mathbf{x}, \mathbf{y}\}$, is

[6] The symmetry axiom refers to symmetric problems with respect to non-EU preferences from $\mathcal{P}^B$, while RST's symmetry relates only to symmetric problems with respect to EU preferences (in which case symmetry with respect to elementary lotteries is equivalent to symmetry with respect to the entire set of lotteries). The reason for the difference is the domain chosen for the bargaining solution. When the domain refers to a fixed alternative set and varying preference relation pairs, consideration of only symmetric EU problems is sufficient for the characterization. In this paper, the domain chosen fixes a possibly non-EU preference relation pair, hence the characterization requires symmetry also with respect to non-EU preferences.

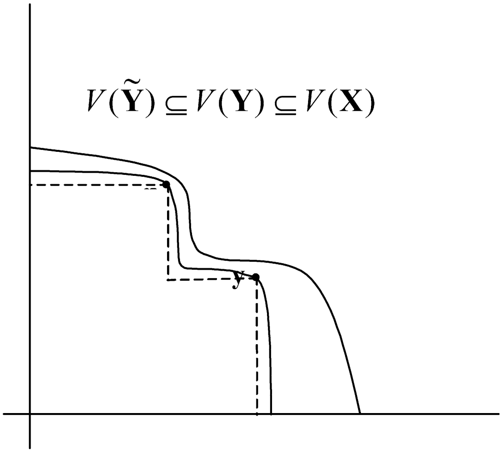

Fig. 3. Existence of $\langle \mathbf{Y}, D\rangle \in \mathcal{B}$.

not a connected set. However, since v_i are continuous functions and $\mathbf{X}$ is D-comprehensive, there exists $\langle \mathbf{Y}, D\rangle \in \mathcal{B}$ such that $\tilde{\mathbf{Y}} \subseteq \mathbf{Y} \subseteq \mathbf{X}$, $\{\mathbf{x}, \mathbf{y}\} \subseteq F(\mathbf{Y}, D)$ and any $(a, b) \in V(\mathbf{Y})$ implies $(b, a) \in V(\mathbf{Y})$ (see Fig. 3). Therefore, under monotonicity of v_i, for every $\mathbf{w} \in \mathbf{Y}$ there exists a unique $\mathbf{z}(\mathbf{w}) \in \mathbf{Y}$ such that for both i and $j \neq i$, $v_i(w_i) = v_j[(\mathbf{z}(\mathbf{w}))_j]$. Define the function $\phi_{\mathbf{Y}} : \mathbf{Y} \to \mathbf{Y}$ such that for every $\mathbf{w} \in \mathbf{Y}$, $\phi_{\mathbf{Y}}(\mathbf{w}) = \mathbf{z}(\mathbf{w})$. Thus $\phi_{\mathbf{Y}}(\mathbf{x}) = \mathbf{y}$, $\phi_{\mathbf{Y}}(D) = D$ and for every $\mathbf{w}, \mathbf{z} \in \mathbf{Y}$, $\phi_{\mathbf{Y}}(\mathbf{w}) = \mathbf{z}$ implies $\phi_{\mathbf{Y}}(\mathbf{z}) = \mathbf{w}$. Moreover for every $p\mathbf{w}, q\mathbf{z}$, $p\mathbf{w} \succsim_1 q\mathbf{z} \Leftrightarrow g(p)v_1(w_1) \geqslant g(q)v_1(z_1) \Leftrightarrow g(p)v_2[\phi_{\mathbf{Y}}(\mathbf{w})] \geqslant g(q)v_2[\phi_{\mathbf{Y}}(\mathbf{z})] \Leftrightarrow p\phi_{\mathbf{Y}}(\mathbf{w}) \succsim_2 q\phi_{\mathbf{Y}}(\mathbf{z})$. Therefore $\langle \mathbf{Y}, D\rangle$ is a symmetric bargaining problem where $\phi_{\mathbf{Y}}$ is the symmetry function. Thus by **SYM**, $\mathbf{x} \in \mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{Y}, D) \Leftrightarrow \mathbf{y} \in \mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{Y}, D)$. If $\mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D) \cap \mathbf{Y} = \emptyset$, then $\mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D) \cap \{\mathbf{x}, \mathbf{y}\} = \emptyset$. If $\mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D) \cap \mathbf{Y} \neq \emptyset$, then by $\mathbf{Y} \subseteq \mathbf{X}$ and **WARP**, $\mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{Y}, D) = \mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D) \cap \mathbf{Y}$, therefore $\mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{Y}, D) \cap \{\mathbf{x}, \mathbf{y}\} = \mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D) \cap \{\mathbf{x}, \mathbf{y}\}$. Hence, in both cases, $\mathbf{x} \in \mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D) \Leftrightarrow \mathbf{y} \in \mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D)$. It follows that for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$,

$$\text{if } \prod_i v_i(x_i) = \prod_i v_i(y_i) \text{ then either } \mathbf{x}, \mathbf{y} \in \mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D) \text{ or } \mathbf{x}, \mathbf{y} \notin \mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D).$$

Suppose now that $\prod_i v_i(x_i) > \prod_i v_i(y_i)$ and let $\mathbf{z} \in \mathbf{X}$ such that $z_i = px_i + (1-p)D_i$ for some $p \in (0, 1)$ and $\prod_i v_i(z_i) = \prod_i v_i(y_i)$, thus $\mathbf{x} > \mathbf{z}$. Then, by **PAR**, $\mathbf{z} \notin \mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D)$ and therefore $\mathbf{y} \notin \mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D)$. Thus $\mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D) \subseteq \arg\max_{\mathbf{x}\in\mathbf{X}} \prod_i v_i(x_i)$. Since $\mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D)$ is always non-empty and for any $\mathbf{z}, \mathbf{w} \in \arg\max_{\mathbf{x}\in\mathbf{X}} \prod_i v_i(x_i)$, $\prod_i v_i(z_i) = \prod_i v_i(w_i)$, it follows that $\mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D) = \arg\max_{\mathbf{x}\in\mathbf{X}} \prod_i v_i(x_i)$. Hence $\mathcal{F}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D) = \mathcal{A}_{\langle \succsim_1, \succsim_2\rangle}(\mathbf{X}, D)$ by Theorem 1. $\square$

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