Nash Bargaining Over Allocations in Inventory Pooling Contracts

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Abstract: When facing uncertain demand, several firms may consider pooling their inventories leading to the emergence of two key contractual issues. How much should each produce or purchase for inventory purposes? How should inventory be allocated when shortages occur to some of the firms? Previously, if the allocations issue was considered, it was undertaken through evaluation of the consequences of an arbitrary priority scheme. We consider both these issues within a Nash bargaining solution (NBS) cooperative framework. The firms may not be risk neutral, hence a nontransferable utility bargaining game is defined. Thus the physical pooling mechanism itself must benefit the firms, even without any monetary transfers. The firms may be asymmetric in the sense of having different unit production costs and unit revenues. Our assumption with respect to shortage allocation is that a firm not suffering from a shortfall, will not be affected by any of the other firms' shortages. For two risk neutral firms, the NBS is shown to award priority on all inventory produced to the firm with higher ratio of unit revenue to unit production cost. Nevertheless, the arrangement is also beneficial for the other firm contributing to the total production. We provide examples of Uniform and Bernoulli demand distributions, for which the problem can be solved analytically. For firms with constant absolute risk aversion, the agreement may not award priority to any firm. Analytically solvable examples allow additional insights, e.g. that higher risk aversion can, for some problem parameters, cause an increase in the sum of quantities produced, which is not the case in a single newsvendor setting.
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1. INTRODUCTION

Inventory pooling among different branches/retailers utilizing a single warehouse under conditions of uncertain demand has been shown to be beneficial for a centralized firm, i.e., under a single decision maker [8, 10]. Pooling has proven beneficial under various operational strategies including delayed differentiation, assemble to order, and substitution. With respect to decentralized systems, a recent, prominently debated issue considered the potential benefits to airlines from pooling their spare parts when utilizing the same type of aircraft and operating a hub at the same airport [5]. In this article we analyze several firms (for example retailers), i.e., multiple decision makers, who are considering pooling inventory. Under the cooperative framework in which these firms agree to share information, the subsequent coordinated behavior will lead to an a-priori efficient agreement. Nevertheless, pursuing their individual interests will result in actions different from those of a centralized system. This gives rise to questions concerning choices drawing from cooperative action and their effects and benefits for each firm individually. In particular, two key contractual issues emerge. One considers how much each should produce/purchase to inventory prior to demand realization. The other considers how to allocate inventory in case of shortage(s) to some of the firms. We envision contracts that will specify both aspects a-priori. Some of these issues were previously analyzed for risk neutral firms with symmetric (i.e., equal across firms) costs in a cooperative setting [12, 13 and references therein] and for unequal shortage costs [18, 1]. But none of these authors considered endogenously determined "entitlements" within a cooperative bargaining setting or unequal production costs or risk aversion. An interesting approach, provided in [4], conducts trading negotiations of initially purchased goods and/or capacity after revising demand forecasts but before actual demand realization. We also note that unlike the stream of research dealing with expected cost core allocations in pooling situations [13, 9], our model allocates physical inventory/shortage (if any) for each demand realization.

Our focus is on a cooperative bargaining solution, reflecting the contractual nature of the pooling arrangement. Consistent with most of the centralized inventory pooling literature, but differing from the decentralized literature cited

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above, we assume that firms cannot compensate each other through monetary transfers. Consequently, all firms must benefit directly from the contract (in expectation). The impossibility of such side payments may be justified by anti-trust regulations, common in many markets. In addition, the decentralized cooperative system we analyze is efficient even without such side payments. Moreover, the firms may not be risk neutral, hence different efficient contracts may provide different sums of utilities to the firms. Such a setting is known in the literature as nontransferable utility (NTU), as opposed to Slikker et al.'s [18] transferable utility (TU) setting.[1] The nonexistence of side payments renders our setting NTU even for risk neutral firms. The model considers several possibly risk averse firms with asymmetric cost structures that sign contracts specifying the quantities to be produced for inventory, as well as each firm's allocations in the case of shortages. We assume that if a firm does not suffer a shortfall, it is not affected by any of the other firms' shortfalls. The firms bargain over these contracts and the outcome is determined by the Nash bargaining solution (NBS [15]). The disagreement point consists of the respective expected utilities in the nonpooled scenario. The bargaining literature has emphasized the disagreement outcome as a crucial factor inducing cooperation, thus all firms must benefit from bargaining compared to the disagreement point. Our work is the first to combine risk aversion and Nash bargaining in an inventory pooling setting [14]. However, our bargaining setting is innovative also in the risk neutral firms scenario because the entitlements are determined endogenously.

The results of this work will emphasize the differences between centralized and cooperative, decentralized systems, despite the fact that we assume that under both systems all players share information. In the centralized system, the single decision maker maximizes the profits of the entire entity (and will only produce in the branch(es) with the lowest operating costs), whereas we find that a cooperative, decentralized system, striving for Pareto optimality, will maximize the sum of "profitability ratios," i.e. the sum of ratios of certainty equivalent (CE) profits to unit production cost (meaning that firms with higher unit operating costs may still contribute). In the risk neutral case, the profitability ratio reduces to the ratio of expected profit to unit production cost. Thus our findings highlight the importance of recognizing whether the setting includes a single or multiple decision makers.

In the next section, we introduce our bargaining over inventory allocations model. Section 3 analyzes the setting in a risk neutral scenario and provides interpretive examples using uniform and Bernoulli demand distributions. We then focus in Section 4 on firms with constant absolute risk aversion (CARA). Among our findings: (1) all firms improve their CE profits when their inventories are pooled according to the NBS, (2) the entitlements of firms do not depend directly on the disagreement point, but instead maximize the sum of ratios of CE profit to unit production cost (this implies that cooperating firms do *not* maximize the sum of their profits in order to determine the quantities to be produced), (3) for two risk neutral firms, one firm receives "absolute priority" on all inventory while the other contributes a positive initial inventory, however both firms still gain from cooperation, (4) under risk neutrality, the expected profit gain with respect to the disagreement outcome is proportional to the firm's production cost, thus the expected gain from cooperation is split equally when the unit production costs are equal, and (5) due to benefits from risk pooling, higher risk aversion can, for some problem parameters, cause an increase in the sum of quantities produced, which is not the case in Eeckhoudt et al.'s [7] single newsvendor setting. In the final section, we discuss some additional issues and generalizations. In particular, we show that for an important class of symmetric joint demand distributions, if the setting is viewed as a TU game, then a coalition's expected profit per firm is increasing in its size. Such a property, in turn, implies that the NBS agreement is in the core of the game.

2. BARGAINING MODEL

We consider n possibly risk averse firms with asymmetric cost structures (unequal unit production costs and unit revenues). The firms bargain over a contract for q_i, the quantity produced or purchased by each firm $i \in \{1,..,n\}$ prior to demand realization, as well as on "entitlements" e_i, where $\sum_i e_i = \sum_i q_i$. The entitlements, a novel concept, determines how inventory is allocated in the case of potential shortages. Considering two firms in the initial case, we assume that after demand is realized, if only one firm has a shortfall relative to its entitlement, the other firm is not assigned any shortage. In the event of both firms having a shortfall, each firm bears its own shortfall. This natural scheme treats the firms in a symmetric, fair approach (pooling is not beneficial in the latter case, so each firm has to bear the shortage by their own). While plausible, that arrangement could in itself be negotiable, but we take it as given. The bargained contract specifies the values of q_i and e_i a-priori for both firms. Denoting realized demand by x_i and the preassigned shortages by $s_i(x, e)$, where[2] $x = (x_i)_i$ and $e = (e_i)_i$, the situation is summarized in Table 1. The first row in the table describes the case where both firms have a shortfall relative to their

[1] Several versions of the NTU Shapley Value exist [11], but are not easy to compute hence are rarely applied.

[2] For any sequence of numbers $t_1, \dots, t_n$, the corresponding vector is denoted by $(t_i)_i$.

Table 1. Two firm allocation.

Shortage contribution	Shortage allocation[a]		Received from other firm	
	$s_1(x,e)$	$s_2(x,e)$	Firm 1	Firm 2
$x_1 > e_1, x_2 > e_2$	$x_1 - e_1$	$x_2 - e_2$	$(e_1 - q_1)^+$	$(e_2 - q_2)^+$
$x_1 > e_1, x_2 \le e_2$	$(x_1 + x_2 - e_1 - e_2)^+$	0	$(q_2 - x_2)^+$	$(x_2 - q_2)^+$
$x_1 \le e_1, x_2 > e_2$	0	$(x_1 + x_2 - e_1 - e_2)^+$	$(x_1 - q_1)^+$	$(q_1 - x_1)^+$

[a] $\max\{y, 0\}$ is denoted by y^+.

respective entitlements. Since each firm bears its own shortage in this case, the shortages equal the difference between realized demand and entitlement, and the demand satisfied equals the firm's entitlement. While we will work with shortage allocations, we also display here a perhaps more intuitive concept, the quantity of inventory received from the other firm after demand realization. In the first row, the quantity each firm receives equals the difference between its entitlement and the quantity it produced (if positive). In the second row of the table, only the first firm suffers a shortfall relative to its entitlement. According to the contract, the second firm bears no shortage and the shortage allocation of the first firm is thus the summed realized demand less summed entitlement (which also equals summed quantity produced) of both firms. Herein lies the advantage of pooling. If the difference between the second firm's production and realized demand is positive, then the first firm receives this difference, as well as its own production. If this difference is negative, then the second firm receives its absolute value. The third row of the table describes a symmetric scenario to the one in the second row.

In the case of more than two firms, the contract must specify the shortage allocation when more than one firm, but not necessarily all of them, experiences a shortfall. This is obtained by considering each firm's shortage relative to e_i and allocating the total shortage proportionally.[3] Define the overall proportionality factor by

$$\gamma(x,e) = \frac{\left[\sum_j (x_j - e_j)\right]^+}{\sum_j (x_j - e_j)^+}.$$

Thus $0 \le \gamma(x,e) \le 1$. Each firm i's shortage is then

$$s_i(x,e) = \gamma(x,e)(x_i - e_i)^+.$$

This allocation scheme reduces to the one defined in Table 1 when there are two firms.

We consider Nash bargaining problems [15], in which firms bargain over inventory pooling contracts. Both the quantities produced and the entitlements are simultaneously determined by the Nash bargaining solution (NBS) where the disagreement point consists of the respective expected utilities in the non-pooled scenario. In other words, NBS maximizes the product of the additional utilities of pooled inventories and entitlements over and above the nonpooled utilities.

We will now discuss some of the basic rationality and fairness concepts that the bargaining scheme ought to reflect [15]. The solution outcome should be Pareto optimal, i.e., there exists no alternative at least as good for all players and better for at least one of them. In cases where the bargaining situation is symmetric, the outcome should be symmetric with respect to the players' identities. The scale of measurement (related by affine transformations) should not affect the solution. Furthermore, the presence of nonselected alternatives should not affect the solution outcome (independence of irrelevant alternatives). Nash proved that the unique solution satisfying these axioms is defined by bargaining outcomes that maximize the product of utility gain beyond the disagreement point, generally referred to as the NBS. It can also be shown that an intuitive alternating offers bargaining scheme leads to the same result as the NBS [2]. A further justification for the NBS is provided in Border and Segal [3], who use ex-ante and ex-post fairness axioms to deduce that NBS is the "best" among all possible bargaining solutions. In the final section we explain why subcoalitions will not emerge in our setting.

Contrary to Slikker et al. [18], side payments are not permitted. We are consistent with most of the centralized inventory pooling literature in ruling out side payments. Although the transformation from quantities produced q_i to entitlements e_i can be viewed as a transfer for every demand realization, it is not like a monetary transfer because of differing production costs and risk aversion. Consequently the setting falls in the category of nontransferable utility (NTU) bargaining games.

A contract consists of a pair (q,e) such that $q = (q_i)_i$, where q_i is an initial inventory level contributed by firm i and e_i is a reference quantity attributed to i such that $\sum_i e_i = \sum_i q_i$. Contingent on demand, shortages $s_i(x,e)$ are allocated only to firms whose demand is greater than their entitlement. Note that shortages are evaluated relative to the e_i's and thus do not directly depend on q.

[3] We allocate proportionally the realized shortage relative to the entitlement. This is different from allocating the expected (shortage and overage) costs, as in [9, 13].

Suppose that firm i has a utility function u_i for money. Firm i's expected (w.r.t. the joint demand distribution) utility given a contract (q, e) is

$$w_i(q, e) = Eu_i[b_i[x_i - s_i(x, e)] - c_i q_i]$$

where $b_i > 0$ is the revenue received per unit and $c_i > 0$ is its unit production cost [7]. We denote $w(q, e) = (w_i(q, e))_i$. The model does not consider initial wealth, because it only addresses linear utilities or utilities with constant absolute risk aversion (CARA).

The disagreement outcome represents the nonpooling situation (q^d, e^d) with utilities $w^d(q^d, e^d)$, where $q_i^d = e_i^d$ is the optimal initial quantity when each firm i operates alone and shortages are handled by each firm separately, i.e., $s_i^d(x, e) = (x_i - e_i^d)^+$. Unless the demands are perfectly positively correlated, there is always room for bargaining because the disagreement outcome achieves a strictly lower expected utility for all firms than the contract (q^d, e^d) with risk pooling according to Table 1.

The NBS outcome is computed by[4]

$$(q^*, e^*) \in \arg \max_{(q,e)|w_i(q,e)\geq w_i^d} \prod_i \left[w_i(q, e) - w_i^d\right]$$

where $\sum_i e_i = \sum_i q_i$, and we denote $w^* = w(q^*, e^*)$. That is, the product of each firm's utility improvement over the nonpooled situation is maximized. Below we refer to $f(q, e)$, the certainty equivalent (CE) profit corresponding to $w(q, e)$, where $f(q, e) = (f_i(q, e))_i$ and $f_i(q, e)$ satisfies $u_i[f_i(q, e)] = w_i(q, e)$. Disagreement CE profit is denoted by f^d.

3. RISK NEUTRALITY

Under risk neutrality, $w_i(q, e) = R_i(e) - O_i(q)$, where $R_i(e) \equiv b_i E[x_i - s_i(x, e)]$ is the expected revenue and $O_i(q) \equiv c_i q_i$ is the production cost. The shortages $s_i(x, e)$, hence $R_i(e)$, are functions of all e_j.

Consider a centralized solution that maximizes the sum of profitability ratios (ratio of profit to unit production cost), where the quantities vector, say y^*, is all that needs to be decided. More specifically, the problem is $y^* \in \arg\max_y \sum_i \frac{1}{ci}[R_i(y) - c_i y_i]$. We now show that in our decentralized bargaining model, the entitlements e^*, that are reached as part of the bargaining outcome, are exactly equal to such centralized quantities y^*. In particular, this implies that cooperating firms do not maximize the sum of their profits in order to determine the quantities produced. This result has the following strong justification. Recall that a bargaining outcome is necessarily Pareto optimal, i.e., there exists no alternative at least as good for all players and better for at least one of them. The following proposition states that for a bargaining outcome to be Pareto optimal, it is necessary and sufficient that the entitlements e^* are equal to such centralized quantities y^*. Recall that $f_i(y, y)$ is the CE profit for firm i when both the quantity produced and the entitlements are the quantity vector y.

PROPOSITION 1: A contract (q, e) is Pareto optimal if, and only if,

$$e \in \arg\max_{e'} \sum_i \frac{1}{c_i}\left[R_i(e') - c_i e_i'\right],$$

and equivalently,

$$e \in \arg\max_{e'} \sum_i \frac{1}{c_i} f_i(e', e').$$

PROOF: A contract (q, e) does not maximize the sum of profitability ratios when there exists $\hat{e}$ satisfying $\sum_i[\frac{1}{c_i}R_i(\hat{e}) - \hat{e}_i] > \sum_i[\frac{1}{c_i}R_i(e) - e_i]$. This holds if, and only if, $\sum_i[\frac{1}{c_i}R_i(\hat{e}) - \hat{e}_i] > \sum_i[\frac{1}{c_i}R_i(e) - q_i]$. The latter occurs if, and only if, there exist $\hat{q}_i$ such that $\sum_i \hat{q}_i = \sum_i \hat{e}_i$ for which $\frac{1}{c_i}R_i(\hat{e}) - \hat{q}_i > \frac{1}{c_i}R_i(e) - q_i$ for all i. This in turn holds if, and only if, $w_i(\hat{q}, \hat{e}) - w_i^d > w_i(q, e) - w_i^d \geq 0$ for all i, i.e. (q, e) is not Pareto optimal. Therefore (q, e) is Pareto optimal if, and only if, e maximizes $\sum_i[\frac{1}{c_i}R_i(e) - e_i]$. The equivalent maximization follows because $w_i(q, e) = f_i(q, e)$ for all (q, e) under risk neutrality. □

An important consequence of Proposition 1 is the fact that for two firms, only the more efficient manufacturer will have any entitlement. The other firm's customers will only be satisfied in case of surpluses. We elaborate on this point in Proposition 2 below. Note that the entitlements e^* do not depend on the disagreement scenario directly (later we show that the quantities produced q^* do depend on the disagreement outcome directly). Instead, the optimal entitlements maximize the sum of "profitability ratios," i.e., the sum of ratios of expected profit to unit production cost.

Proposition 1 implies the first order conditions

$$\sum_i \frac{1}{c_i} \frac{\partial R_i(e)}{\partial e_j} = 1, \quad \text{for all } j.$$

[4] According to the NBS, a feasible alternative is a lottery of contracts, yielding the contracts $(q^1, e^1), \ldots, (q^l, e^l)$ with probabilities $p^1, \ldots, p^l$ respectively, for some $l \geq 1$ such that $p^1 + \cdots + p^l = 1$. Assuming risk neutrality or aversion, $l = 1$ for Pareto optimal lotteries. Thus without loss of generality, since a bargaining outcome is always Pareto optimal, we can avoid randomizations of contracts (q, e) in our model.

3.1. Two Firms

When there are two firms, we can assume without loss of generality that $\frac{b_1}{c_1} \geq \frac{b_2}{c_2}$. Under separate operations with identical demand distributions, this implies that firm 1 will stock at least as much as firm 2.

It should be noted that if $e_i = 0$, firm j is entitled to "absolute priority." This is because firm i's zero entitlement means that firm j is entitled to the total quantity produced. In this case, only after satisfying firm j's demand, may the remaining quantity be used to satisfy firm i's demand.

PROPOSITION 2: There always exists e^* satisfying $e_2^* = 0$, i.e. the firm with the higher $\frac{b_i}{c_i}$ receives "absolute priority."

PROOF: Using Proposition 1, we can compute $\max_{e'} \sum_i \frac{1}{c_i}[R_i(e') - c_i e'_i]$ by first fixing $\sum_i e'_i = z$ and then maximizing over z. Denoting $\alpha(z) = \max_{e' | \sum_i e'_i = z} \sum_i \frac{1}{c_i} R_i(e')$, we have $\max_{e'} \sum_i [\frac{1}{c_i} R_i(e') - e'_i] = \max_z[\alpha(z) - z]$. Then for any z, $\alpha(z) = \sum_i \frac{b_i}{c_i} E[x_i - s_i(x, (z, 0))]$, because $(z, 0)$ is a maximizer for every realization of x. This establishes the absolute priority of firm 1. □

Although firm 1 receives absolute priority, firm 2 may produce a positive quantity q_2^* and still gain from cooperation. Note that we derive this absolute priority from NBS (contrary to [1], where it is essentially given exogenously).

Given that $e_2^* = 0$ by Proposition 2, the first order conditions can be used to determine e_1^* by solving $\sum_i \frac{1}{c_i} \frac{\partial R_i(e)}{\partial e_1} = 1$ under the condition $e_2 = 0$. To compute q^* (after we computed e^*), we solve $\max_{q1} \prod_i [w_i(q, e^*) - w_i^d]$ subject to $w_i(q, e^*) \geq w_i^d$ and $q_2 = e_1^* - q_1$. Thus the first order condition gives $q_i^* = \frac{1}{2}[e_1^* + \frac{1}{c_i}[R_i(e^*) - w_i^d] - \frac{1}{c_{-i}}[R_{-i}(e^*) - w_{-i}^d]]$ ($-i$ denotes the firm other than i). Finally, the efficient frontier of the bargaining problem is linear in expected utilities, thus the solution outcome is its mid-point and we have

$$w_i^* = w_i^d + \frac{c_i}{2} \sum_j \frac{1}{c_j} \left[w_j(e^*, e^*) - w_j^d \right].$$

The optimal quantities produced can then be computed by $q_i^* = \frac{1}{c_i}[R_i(e^*) - w_i^*]$. Note that under identical unit production costs $c_1 = c_2$, the gain from cooperation is split equally, thus the outcome w^* is the TU Shapley value [17]. But if $c_1 \neq c_2$, the NBS approach is essential.

When $\frac{b_1}{c_1} = \frac{b_2}{c_2}$ there are many NBS outcomes with varying priority schemes, including absolute priority to firm 2 and equal priority to both firms (e.g., $e_1^* = e_2^*$ under symmetric scenarios). All these outcomes have the same sum of quantities produced but varying allocations of this sum, so that the higher e_i^* for firm i, the higher is q_i^* for this firm. In other words, a firm that receives low priority is compensated by contributing a lower portion to the overall quantity produced. Both firms are indifferent between all these NBS outcomes.

3.2. Illustration: Two Firms Faced With Independent Uniform Demand Distributions

Suppose that x_i are uniformly and independently distributed over [0,1]. Assuming that the two firms have reached agreement according to NBS, since $e_1 \leq 2$ by assumption with respect to demand and since $e_2 = 0$ as shown in Section 3.1,

$$\begin{aligned}
&\sum_i \left[\frac{1}{c_i} R_i(e) - e_i \right] \\
&= \frac{1}{2} \sum_i \frac{b_i}{c_i} - \sum_i \frac{b_i}{c_i} \int_{\min\{1, e_{-i}\}}^1 \int_{\min\{1, e_i\}}^1 (x_i - e_i) dx_i dx_{-i} \\
&\quad - \frac{b_2}{c_2} \int_{(e_1-1)^+}^{\min\{1, e_1\}} \int_{e_1 - x_1}^1 (x_1 + x_2 - e_1) dx_2 dx_1 - \sum_i e_i \\
&= \begin{cases}
\frac{1}{2} \sum_i \frac{b_i}{c_i} - \frac{b_1}{2c_1}(1 - 2e_1 + (e_1)^2) \\
\quad - \frac{b_2}{2c_2} \left(1 - (e_1)^2 + \frac{1}{3}(e_1)^3 \right) - e_1 & \text{if } e_1 \in [0,1] \\
\frac{1}{2} \sum_i \frac{b_i}{c_i} - \frac{b_2}{c_2} \left((e_1)^2 - 2e_1 - \frac{1}{6}(e_1)^3 + \frac{4}{3} \right) - e_1 \\
& \text{if } e_1 \in [1,2]
\end{cases}
\end{aligned}$$

Denoting $\frac{b_i}{c_i}$ by r_i, the first order condition implies

$$e_1^* = \begin{cases} 1 - \frac{r_1}{r_2} + \sqrt{1 - \frac{2}{r_2} + \left(\frac{r_1}{r_2} \right)^2} & \text{if } e_1^* \leq 1 \\ 2 - \sqrt{\frac{2}{r_2}} & \text{if } e_1^* \geq 1. \end{cases}$$

In disagreement, $q_i^d = e_i^d = (1 - \frac{c_i}{b_i})^+$ and

$$\begin{aligned}
f_i^d = w_i^d &= \frac{b_i}{2} - b_i \int_{q_i^d}^1 \left(x_i - q_i^d \right) dx_i - c_i q_i^d \\
&= (b_i - c_i) q_i^d - \frac{b_i}{2} \left(q_i^d \right)^2
\end{aligned}$$

which equals $\frac{b_i}{2}(1 - \frac{c_i}{b_i})^2$ for $b_i > c_i$.

The NBS CE profit is $f_i^* = w_i^* = w_i^d + \frac{c_i}{2}[\sum_j (\frac{1}{c_j} R_j(e^*) - e_j^*) - \sum_j \frac{1}{c_j} w_j^d]$ and the quantities produced are $q_i^* = \frac{1}{c_i}[R_i(e^*) - w_i^*]$, thus

$$q^* = \begin{cases}
\Big(\frac{r_1}{2} - \frac{r_1}{2}\left(1 - 2e_1^* + (e_1^*)^2\right) - \frac{1}{c_1} w_1^*, \\
\quad \frac{r_2}{2} - \frac{r_2}{2} \left(1 - \left(e_1^*\right)^2 + \frac{1}{3}\left(e_1^*\right)^3 \right) - \frac{1}{c_2} w_2^* \Big) & \text{if } e_1^* \leq 1 \\
\Big(\frac{r_1}{2} - \frac{1}{c_1} w_1^*, \\
\quad \frac{r_2}{2} - r_2 \left(\left(e_1^*\right)^2 - 2e_1^* - \frac{1}{6}\left(e_1^*\right)^3 + \frac{4}{3} \right) - \frac{1}{c_2} w_2^* \Big) \\
& \text{if } e_1^* \geq 1.
\end{cases}$$

EXAMPLE 1: Suppose there are two firms, the first with lower revenue per unit, $b_1 = 3$, than the second firm, $b_2 = 4$, under the assumption that the firms sell in separate markets. Suppose further that the firms have different production technologies, so that the first has a lower unit production cost, $c_1 = 1$, than that of the second firm, $c_2 = 2$. According to the formulae specified in this section, under disagreement $f^d = w^d = q^d = e^d = (\frac{2}{3}, \frac{1}{2})$ and the NBS cooperative outcome leads to a CE profit $f^* = w^* = (\frac{19}{24}, \frac{3}{4})$, achieved when $e^* = (1, 0)$ and $q^* = (\frac{17}{24}, \frac{7}{24})$. Although firm 2 contributes a positive initial inventory ($q_2^* = \frac{7}{24}$) shortages are always fully charged to this firm due to its lower ratio of expected profit and unit production cost. Nevertheless, both firms gain CE profits with respect to the disagreement outcome and their gains are proportional to c_i. Note that firm 2 may sell more or less than q_2^*, depending on the amount received after firm 1's demand is satisfied. In particular, firm 2 produces less than when operating alone ($\frac{7}{24} < \frac{1}{2}$), but satisfies all of its demand with probability $\frac{1}{2}$ (when total demand is less than total quantity produced), even when its demand is higher than the quantity produced when operating alone. For comparison, if $b_2 = 3$ then $e^* = (0.91485, 0)$ and if $b_2 = 10$ then $e^* = (0, 1.1835)$. In other words, as the firm becomes relatively less profitable, its entitlement drops and vice versa.

3.3. Illustration: Two Firms Faced With Independent Bernoulli Demand Distributions

Suppose that each x_i is distributed independently, having value one with probability p_i and value 0 otherwise. In other words, positive demand is realized or not. Customers ordering one unit are assumed to be willing to purchase any fraction between 0 and 1. This scenario applies, for example, when each firm has a customer who places an order for a component only if her own customer places an order of some fixed size with her. Consequently the firm exhibits demand for one unit with some probability, any available portion of which will be purchased.

Under the disagreement outcome, $q_i^d = I\{p_i b_i \geq c_i\}$, i.e., a firm will produce one unit if $p_i b_i \geq c_i$, otherwise the firm will not produce. This reflects a necessary and sufficient condition for profitability, because $p_i b_i$ is the expected revenue and c_i is the production cost.

Under agreement (NBS), since $e_1 \leq 2$ based on assumed demand and $e_2 = 0$ as shown in Section 3.1,

$$\sum_i \left[\frac{1}{c_i} R_i(e) - e_i\right] = \left\{\begin{matrix} r_1 p_1 e_1 + r_2(1 - p_1) p_2 e_1 - e_1 \\ \text{if } e_1 \in [0, 1] \\ r_1 p_1 + r_2 p_2 [p_1(e_1 - 1) \\ + (1 - p_1)] - e_1 \quad \text{if } e_1 \in [1, 2] \end{matrix}\right\}.$$

This sum is concave[5] and piecewise linear as a function of e_1 over [0, 2] with a "break" at $e_1 = 1$. Therefore the optimal e_1^* implies producing two units when $r_2 p_1 p_2 \geq 1$, producing one unit when $r_1 p_1 + r_2(1 - p_1) p_2 \geq 1$, otherwise there is no production. Consequently production occurs under $r_1 p_1 + r_2(1 - p_1) p_2 \geq 1$, a condition implied by the corresponding one under disagreement, $p_i r_i \geq 1$ for both i, but not vice versa. Thus, cooperation can turn two unprofitable firms into profitable firms. For example, consider $p_i = \frac{1}{4}$, $b_i = 3$, and $c_i = 1$ for both firms. In disagreement, $f_i^d = w_i^d = q_i^d = 0$. However, $e^* = (1, 0)$ and $q^* = (\frac{19}{32}, \frac{13}{32})$ is an NBS outcome, for which the CE profit to each firm is $f_i^* = w_i^* = \frac{5}{32} > 0$, thus both firms gain from cooperation.

4. CONSTANT ABSOLUTE RISK AVERSION

We now consider risk averse firms, but of a particular kind (more general analysis is provided in the Appendix). Under constant absolute risk aversion (CARA), firms behave according to a negative exponential utility function $u_i(t) = -\exp(-k_i t)$, $k_i > 0$. Let $R_i(e) = [E e^{-k_i b_i [x_i - s_i(x,e)]}]^{-1}$ and $O_i(q) = e^{k_i c_i q_i}$. Then,

$$w_i(q, e) = -E \exp\{-k_i b_i [x_i - s_i(x, e)] + k_i c_i q_i\} = -\frac{O_i(e)}{R_i(q)}.$$

Note that this expected utility is increasing in revenue and decreasing in costs (but in ratio form, rather than in the additive form of risk neutral firms).

PROPOSITION 3: A contract (q, e) is Pareto optimal if, and only if,

$$e \in \arg\max_{e'} \sum_i \left[\frac{1}{k_i c_i} \ln R_i(e') - e_i'\right],$$

and equivalently,

$$e \in \arg\max_{e'} \sum_i \frac{1}{c_i} f_i(e', e').$$

PROOF: A contract (q, e) does not maximize the sum of profitability ratios when there exists $\hat{e}$ satisfying $\sum_i [\frac{1}{k_i c_i} \ln R_i(\hat{e}) - \hat{e}_i] > \sum_i [\frac{1}{k_i c_i} \ln R_i(e) - e_i]$. This holds if, and only if, $\sum_i [\frac{1}{k_i c_i} \ln R_i(\hat{e}) - \hat{e}_i] > \sum_i [\frac{1}{k_i c_i} \ln R_i(e) - q_i]$. The latter occurs if, and only if, there exist $\hat{q}_i$ such that $\sum_i \hat{q}_i = \sum_i \hat{e}_i$ for which $\frac{1}{k_i c_i} \ln R_i(\hat{e}) - \hat{q}_i > \frac{1}{k_i c_i} \ln R_i(e) - q_i$ for all i. This in turn holds if, and only if, $w_i(\hat{q}, \hat{e}) - w_i^d >$

[5] Concavity follows since the difference in derivatives between the two linear parts is $[r_1 p_1 + r_2(1 - p_1) p_2 - 1] - [r_2 p_1 p_2 - 1] = r_2 p_1 p_2 (\frac{r_1}{r_2} \frac{1}{p_2} + \frac{1}{p_1} - 2) \geq r_2 p_1 p_2 (\frac{1}{p_2} + \frac{1}{p_1} - 2) \geq 0$.

$w_i(q,e) - w_i^d \geq 0$ for all i, i.e., (q,e) is not Pareto optimal. Therefore (q,e) is Pareto optimal if, and only if, e maximizes $\sum_i[\frac{1}{k_i c_i}\ln R_i(e) - e_i]$. The equivalent maximization follows because the CE profits $f_i(q,e) = \frac{1}{k_i}\ln R_i(q) - c_i q_i$ for all (q,e). □

The entitlements e^* have the same form as in the risk neutral case in terms of the CE profit, but a different form in terms of utilities. Under CARA, as before, the entitlements do not depend on the disagreement scenario directly. Instead, as in the risk neutral case, the optimal entitlements maximize the sum of "profitability ratios," i.e., the sum of ratios of CE profit to unit production cost. Note that comparison to the TU Shapley value is not meaningful here, because this concept is not defined for risk averse firms.

Proposition 3 implies the first order conditions

$$\sum_i \frac{\partial R_i(e)/\partial e_j}{k_i c_i R_i(e)} = 1, \quad \text{for all } j.$$

4.1. Illustration: Firms Faced With Independent Bernoulli Demand Distributions

Suppose again that each x_i is distributed independently, having value one with probability p_i and value 0 otherwise and customers ordering one unit are assumed to be willing to purchase any fraction between 0 and 1. While a single risk neutral firm would produce 0 or 1, this may not be the case for a risk averse firm. Under NBS, when $0 \leq \sum_j e_j \leq 2$,

$$R_i(e) = [p_i p_{-i} e^{-k_i b_i e_i} + p_i(1-p_{-i})e^{-k_i b_i \min\{1, e_1+e_2\}} + (1-p_i)]^{-1}.$$

We can solve for e^* by taking the maximum over several cases, depending on the total amount of quantities produced. Here we show one of these cases, in which the total quantity produced is greater than 1, while other cases appear in the Appendix.

CASE 1: Suppose that $1 < e_1^* + e_2^* < 2$, i.e. there is sufficient inventory for one firm but not for both. The first order conditions are $\frac{r_i p_i p_{-i} e^{-k_i b_i e_i}}{p_i p_{-i} e^{-k_i b_i e_i} + p_i(1-p_{-i})e^{-k_i b_i} + (1-p_i)} = 1$, so

$$e_i^* = \min\left[1, \left(\frac{1}{k_i b_i}\ln\frac{r_i - 1}{\left(\frac{1}{p_i}-1\right)\frac{1}{p_{-i}} + \left(\frac{1}{p_{-i}}-1\right)e^{-k_i b_i}}\right)^+\right].$$

Note that the expression above may result in both firms having $0 < e_i^* < 1$, in which case there is no firm with absolute priority.

EXAMPLE 2: Suppose that $p_1 = p_2 = 0.8$, $b_1 = 1.6$, $b_2 = 2$, $c_1 = 1$, $c_2 = 1.3$ and the absolute risk aversion measures are $k_1 = k_2 = 0.1$. According to NBS, the entitlements are $e^* = (0.82\bar{8}, 0.2)$. Consider now an increase in firm 2's risk aversion measure from $k_2 = 0.1$ to $k_2 = 0.3$. The NBS entitlements are now $e^* = (0.82\bar{8}, 0.3)$. This demonstrates that higher risk aversion can cause an increase in the sum of quantities produced, as opposed to the case of a single newsvendor, where it decreases [7]. Intuitively, there is a trade-off between the tendency to produce less when k_i increases, as in the single newsvendor scenario, and the tendency to produce more because of the benefits from risk pooling. The example demonstrates that this trade-off may lead to an overall increase in the quantity produced the greater the risk aversion demonstrated.

In disagreement, $q_i^d = \min\{1, [\frac{1}{k_i b_i}\ln(\frac{r_i - 1}{\frac{1}{p_i}-1})]^+\}$ and $w_i^d = -e^{k_i c_i q_i^d}[p_i e^{-k_i b_i q_i^d} + (1-p_i)]$. In order to compute q^*, we solve $\max_{q_1}\prod_i[w_i(q,e^*) - w_i^d]$ subject to $w_i(q,e^*) \geq w_i^d$ and $q_2 = \sum_i e_i^* - q_1$. Thus the first order condition gives

$$q_1^* = \frac{1}{k_1 c_1}\ln\left[-w_1^* R_1(e^*)\right], \quad \text{where } w_1^* \text{ solves}$$

$$R_1(e^*)^{\frac{k_2 c_2}{k_1 c_1}} R_2(e^*) e^{-k_2 c_2\left(e_1^*+e_2^*\right)} w_2^d\left(-w_1^*\right)^{1+\frac{k_2 c_2}{k_1 c_1}} - \left(1 - \frac{k_2 c_2}{k_1 c_1}\right) w_1^* - \frac{k_2 c_2}{k_1 c_1} w_1^d = 0.$$

Notice that changes in firm i's absolute risk aversion measure k_i affects the bargaining outcome via changes in the firm's utility function, as well as in two other indirect and possibly opposing ways: via changes in entitlements e^* and via changes in the firm's disagreement utility w_i^d. Consequently, we are not able to use existing results in the literature or some other means in order to conclude the overall effect of changes in k_i on the firms' CE profit in general.

EXAMPLE 3: Consider $p_1 = \frac{3}{4}$ and $p_2 = \frac{2}{5}$, $b_1 = 3$, $b_2 = 4$, $c_1 = 1$, $c_2 = 2$ and assume risk aversion with $k_1 = 1$ and $k_2 = \frac{1}{2}$. In disagreement, $q^d = (0.59\bar{7}, 0)$ and the CE profit is $f^d = (0.38\bar{4}, 0)$. The NBS outcome computes $q^* = (0.57\bar{7}, 0.13\bar{3})$, a higher production summation than that in disagreement. Furthermore, $e^* = (0.53\bar{1}, 0.18\bar{0})$ and the CE profit is $f^* = (0.43\bar{2}, 0.09\bar{7})$, i.e., again higher than that achieved under disagreement. The first firm exhibits higher risk aversion, produces less but gains more from cooperation. The second firm only produces after reaching agreement and thus increases its expected profits from nothing to a strictly positive value, despite its lower level of profitability and expected ability to sell.

We conclude with an example assuming several symmetric firms with identical revenue, cost, demand distributions and degrees of risk aversion.

EXAMPLE 4: Suppose that for all i, $b_i = b$, $c_i = c$, $p_i = p$, and $k_i = k$. All firms gain from cooperation and reach agreement according to NBS. Under symmetry, $e_i^* = q_i^* = e_j^*$ for all i, j. Solving for e^*, we compute the maximum of $\frac{1}{k_i c_i} \ln R_i(e) - e_i$. There are several possible outcomes, as in the case of two firms, depending on the total amount of quantities produced. We show here only one case, in which $\frac{n-1}{n} < e_i^* < 1$ for all firms, i.e., there is sufficient inventory for $n-1$ firms but not for all. Then $R_i(e) = [p^n e^{-kbe_i} + p(1-p^{n-1})e^{-kb} + (1-p)]^{-1}$. The first order conditions are $\frac{rp^n e^{-kbe_i}}{p^n e^{-kbe_i} + p(1-p^{n-1})e^{-kb} + (1-p)} = 1$, so

$$\begin{aligned} q_i^* &= e_i^* \\ &= \min\left[1, \left(\frac{1}{kb} \ln \frac{r-1}{\left(\frac{1}{p}-1\right)\frac{1}{p^{n-1}} + \left(\frac{1}{p^{n-1}}-1\right)e^{-kb}}\right)^+\right], \end{aligned}$$

which is quite similar to the allotment of two asymmetric firms and moreover provides the quantities that each firm will produce.

5. SUMMARY, EXTENSIONS, AND FUTURE RESEARCH

This article makes several contributions to multifirm inventory pooling and thus to decentralized OM. We have introduced the concept of entitlements, which formalizes previous priority notions. The entitlements, as well as quantities produced or purchased, are chosen based on the Nash bargaining solution, rarely used in OM, but which appears to be a natural framework for this setting. Allowing for risk averse preferences has led to a scenario without transferable utility.

In the risk neutral domain, addressed by most inventory pooling literature, our "entitlement" framework justifies the use of absolute priority, the first time such a policy is derived from first principles. Moreover, our framework shows that for risk averse firms, absolute priority may not be desirable. The impact of risk aversion may lead to higher or lower levels of production in comparison to the disagreement outcome, a matter not previously discussed in the literature. The trade-off lies between the tendency to produce less when becoming more risk averse, as in the single newsvendor scenario, and the tendency to produce more because of the benefits from risk pooling.

5.1. Sub-Coalitions

One may ask whether the NBS agreement is stable under potential deviations by sub-coalitions. This is especially interesting in cases that may be described as TU games for which an allocation in the core is sought. We now show that for such cases with symmetric multivariate normal demand (at the least), risk pooling advantages are enhanced as the group of contracting firms becomes larger, implying that the NBS outcome of the grand coalition is in the core. Assume n identical risk neutral firms with unit revenue b and unit cost c, such that each firm faces demand distributed $N(m, v^2)$ with $v \ll m$ and correlation coefficient ρ between each pair. Consequently, the total demand for n firms is distributed $N[\mu(n), \sigma^2(n)]$ with $\mu(n) = nm$ and $\sigma^2(n) = v^2 n[1+(n-1)\rho]$, where $n \le 1 - \frac{1}{\rho}$ if $-1 \le \rho < 0$ and n is unbounded otherwise. The optimal total order quantity is $Q^*(n) = \Phi^{-1}(1-\frac{c}{b})v\sqrt{n[1+(n-1)\rho]} + nm$, where Φ is the standard normal cumulative distribution function and Φ^{-1} denotes its inverse function. Computing the optimal expected profit [8] and dividing by the number of firms, the optimal expected profit per firm, $\pi^*(n)$, is

$$\begin{aligned} \pi^*(n) &= \frac{1}{n}\left[b\left(\int_{-\infty}^{Q^*(n)} \frac{x}{\sqrt{2\pi}\sigma(n)} e^{-\frac{1}{2}\left(\frac{x-\mu(n)}{\sigma(n)}\right)^2} dx \right.\right. \\ &\quad \left.\left. + \int_{Q^*(n)}^{\infty} \frac{Q^*(n)}{\sqrt{2\pi}\sigma(n)} e^{-\frac{1}{2}\left(\frac{x-\mu(n)}{\sigma(n)}\right)^2} dx\right) - cQ^*(n)\right] \\ &= \frac{1}{n}\left[b\left([\mu(n) - Q^*(n)]\Phi\left[\frac{Q^*(n)-\mu(n)}{\sigma(n)}\right]\right.\right. \\ &\quad \left.\left. - \frac{\sigma(n)}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{Q^*(n)-\mu(n)}{\sigma(n)}\right)^2} + Q^*(n)\right) - cQ^*(n)\right] \\ &= \frac{1}{n}\left[nm(b-c) - b\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\Phi^{-1}(1-\frac{c}{b}))^2}\right. \\ &\quad \left. \times v\sqrt{n[1+(n-1)\rho]}\right] \\ &= m(b-c) - b\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\Phi^{-1}(1-\frac{c}{b}))^2} v\sqrt{\rho + \frac{1}{n}(1-\rho)}. \end{aligned}$$

Clearly, $\pi^*(n)$ is an increasing function for all n. This observation was also made in [6]. For us, it implies that the NBS outcome is in the core for all n, because any coalition of s firms achieves $s\pi^*(s)$ for its members when operating outside the grand coalition of n firms, which is less than $s\pi^*(n)$ that could be achieved by the firms in that coalition according to NBS in the grand coalition. In fact, the other direction also holds: if the NBS outcome is in the core for all n then $\pi^*(n)$ is increasing for all n. To see this, suppose $\pi^*(s) > \pi^*(n)$ for some $s < n$, then $s\pi^*(s) > s\pi^*(n)$, implying that a coalition of s firms can achieve more than the NBS outcome of the grand coalition of n firms.

Does membership in the core extend to non-symmetric demand scenarios? Consider the following counter example, which adapts Example 3.1 in [13] to normal demand distribution. There are 3 firms with unit revenue $b = 2$, unit cost $c = 1$ and demand such that x_1 is distributed $N(100, 1)$ and the probability $P\{x_1 = x_2 = 200 - x_3\} = 1$. The grand coalition, facing a demand distribution $N(300, 1^2)$, orders total quantity $Q^*_{\{1,2,3\}} = \Phi^{-1}(\frac{1}{2}) + 300$ and achieves expected profit per firm of $\frac{1}{n}[(b-c)\mu - b - \frac{1}{\sqrt{2\pi}}\sigma] = \frac{1}{3}[(2-1)300 - 2\frac{1}{\sqrt{2\pi}}1] \cong 99.7$. However, the coalition of firms $\{1, 3\}$, facing certain demand of 200, orders total quantity 200 and achieves total expected profit $200 > 2 \cdot 99.7 \cong 199.4$. We conclude that although the grand coalition may not be stable, some coalitions of at least two firms will be stable even in such extremely non-symmetric demand scenarios. Our model predicts the settlement reached by such groups of firms. In most practical situations, the increase in the number of firms would be characterized by less extreme non-symmetry, thus leading to larger stable coalitions.

5.2. Salvage Values

If salvage values were present, a_i per unit left at firm i, the expected revenue defined at the beginning of Section 3 would be modified to $R_i(e) \equiv E[b_i l_i(x, e) + a_i(e_i - l_i(x, e))]$, where $l_i(x, e) \equiv x_i - s_i(x, e)$ indicates the sales quantity. Because of the similar structure of this function, Proposition 1 would still hold, i.e., the entitlements would be determined independently of the production quantities and would maximize the sum of modified profitability ratios. We are not sure whether absolute priority would still be the result of NBS under risk neutrality.

5.3. Future Research

Potential directions may be to include transshipment costs as in the decentralized noncooperative game transshipment literature [16]. Since transshipment costs were not included within the model defined here, there was no need to describe a physical or geographical system. Furthermore, if the firms automatically deposit the product in a common warehouse prior to the realization of demand (a standard assumption in the inventory pooling literature), transshipment may be rolled into the production cost. On the other hand, the formulation would need to be adapted were the units to remain at each firm's warehouse until needed and were transshipment costs dependent on the origins and destinations.

APPENDIX

Additional Cases for Two Firms Faced With Bernoulli Demand Distributions

CASE 2: Suppose that $e_1^* + e_2^* = 1$. The first order conditions are $1 - \frac{r_i p_i p_{-i} e^{-k_i b_i e_i}}{p_i p_{-i} e^{-k_i b_i e_i} + p_i(1-p_{-i})e^{-k_i b_i} + (1-p_i)} + \lambda = 0$, where λ is the multiplier of the constraint $e_1 + e_2 = 1$. Thus we have

$$e_1^* = \min\left\{1, \left(\frac{1}{k_1 b_1}\ln z\right)^+\right\}, \text{ where } z \text{ solves}$$

$$\frac{r_1}{r_2} z^{-\frac{k_2 b_2}{k_1 b_1}} \left[\frac{1}{p_1} - 1 + \frac{1}{p_1}\left(\frac{1}{p_2} - 1\right)e^{k_2 b_2}\right]$$

$$-\left[\frac{1}{p_2}\left(\frac{1}{p_1} - 1\right) + \left(\frac{1}{p_2} - 1\right)e^{-k_1 b_1}\right] z + \frac{r_1}{r_2} - 1 = 0.$$

CASE 3: Suppose that $0 < e_1^* + e_2^* < 1$. The first order conditions give

$$e_i^* = \frac{1}{k_i b_i}\ln z_i, \text{ where } z_i \text{ solve the system}$$

$$1 = \frac{r_i}{1 + \left(\frac{1}{p_i} - 1\right)\frac{z_i}{p_{-i} + (1-p_{-i}) z_{-i}^{-\frac{k_i b_i}{k_{-i} b_{-i}}}}} + \frac{r_{-i}}{1 + \frac{1}{1-p_i}\left[p_i + \left(\frac{1}{p_{-i}} - 1\right) z_{-i}\right] z_i^{\frac{k_{-i} b_{-i}}{k_i b_i}}}$$

for three subcases: (i) $e_1^*, e_2^* > 0$, (ii) $e_1^* > 0, e_2^* = 0$ and (iii) $e_1^* = 0$, $e_2^* > 0$.

CASE 4: $e_1^* = e_2^* = 0$. Then $O_i(q^*) = R_i(e^*) = 1$.

First Order Conditions for the General Case

Assume that w_i is differentiable. In general, the solution outcome satisfies the following first order conditions for every j,

$$-\lambda + \frac{\partial w_j(q, e)}{\partial q_j} \prod_{i \neq j} \left[w_i(q, e) - w_i^d\right] = 0$$

$$\lambda + \sum_i \frac{\partial w_i(q, e)}{\partial e_j} \prod_{k \neq i} \left[w_k(q, e) - w_k^d\right] = 0.$$

where λ is the multiplier of the constraint $\sum_i e_i - \sum_i q_i = 0$. In addition, there are only non-negativity constraints. Rewriting the first equation as $\prod_{i \neq j}[w_i(q, e) - w_i^d] = \frac{\lambda}{\partial w_i(q,e)/\partial q_j}$, substituting into the second and cancelling terms, we have for each j,

$$1 + \sum_i \frac{\partial w_i(q, e)/\partial e_j}{\partial w_i(q, e)/\partial q_i} = 0.$$

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